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DEVELOPMENT AND MATHEMATICAL MODELLING OF PLANNING TRAJECTORY OF UNMANNED SURFACE VEHICLE

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Abstract

Marine vessels present dynamic systems which are difficult to control due to their complexity, hydrodynamic effects and external forces. This paper represents a dynamic method which combines trajectory planning, coordinate control and tacking of unmanned surface vessels. It is based on non linear sliding mode control, where the initial trajectory and resultant trajectory is co-linked using a set of ordinary differential equations as position feedback indicators. It is based on the manipulations of linear matrix inequalities (LMIs) imposed by the design objectives. For this purpose, a newly propose LMI conditions for the quadratic performance optimization and the pole-clustering problem, respectively, in a full order state. A control law is developed by introducing a first order sliding surface in terms of surge tracking errors and a second order one in terms of lateral motion tracking errors and its resultant guarantees position tracking while the rotational motion remains bounded. It is implemented for trajectory tracking of under-actuated surface vessels. Control parameters are chosen based on the dynamics of the vehicle, which can change abruptly and significantly when an USV is jettisoned or docked. Tight control of the vehicle state is required to conduct an underway recovery, with looser requirements for the launch. An effort is made to reduce the generic conservatism by allowing different Lyapunov matrices and guarantees convergence and stability under external disturbances. The modelling and identification objective is to determine a model which is sufficiently rich to enable effective model-based control design and trajectory optimization, sufficiently simple to allow parameter identification.

Index Terms: *Surge, Sway, Yaw, Lyapunov equation, Vessels, Earth Fixed Frame, Inertial Frame.*

1. INTRODUCTION

Marine robotics in general represents an interesting and challenging area where the application of control theory presents an essential part. It comes as a consequence of harsh environment in which marine vehicles operate, characterised with unpredictable disturbances. Unmanned robots have a critical role during the investigation of the paths through sign of life. Converted manned vessels have the advantage of accommodation for occupants, which can be used in the preliminary stages of control and development.

2. DESIGN

Mathematical model of a marine platform, defined using two coordinate frame and an Earth fixed (inertial) frame {E} described with axes N, E and D; a body fixed coordinate system {B}, which is usually attached to the centre of gravity of the vehicle and is described with three axis x, y and z pointing respectively in the same directions as the NED frame when x and N are aligned. The mathematical model of a marine platform is described with an assumption that the platform is moving only in the horizontal plane. The platform's speeds are defined in the fixed coordinate frame {B}, surge 'u' and sway 'v' speeds are translation speeds in the x and y direction, and yaw speed 'r' is rotational speed around z direction. Earth fixed coordinate frame is used to define positions x and y in the horizontal plane and orientation of the platform. The motion of the platform is achieved by applying surge (X) and sway (Y) force and yaw (N) moment.

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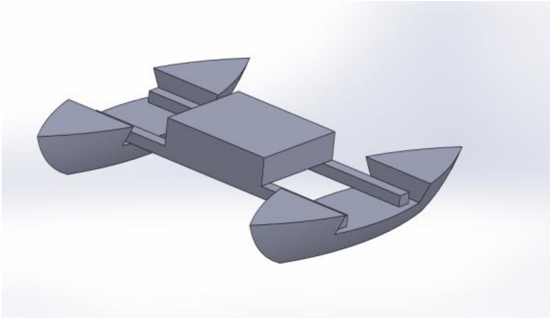

Fig 1: 3-D Concept of the Vehicle

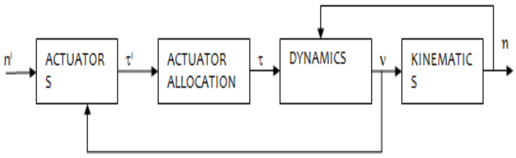

Fig. 2: Block diagram of the mathematical model

2.1. KINEMATIC-DYNAMIC MODEL

The kinematic model in the horizontal plane

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} u \\ v \\ r \end{vmatrix}$$

The dynamic model is assumed to be uncoupled so each controllable degree of freedom (DOF) can be modelled separately. In addition to that, since the platform is symmetric in the horizontal plane, the same model can be used to describe surge and sway dynamics as

$\alpha_u r + \beta\,(u)u = \tau_u E + X$

$\alpha_u r + \beta\,(v)v = \tau_v E + Y$

Where α is a constant parameter and β (u) and β (v) are drag parameter which are speed dependant and include all speeds, the drag parameter can be approximated with a constant term, i.e. $\beta\,(u) = \beta\,(v) = \beta_u$.

Yaw model is given as

$\alpha_r r + \beta\,(r)r = \tau_r E + N$

Where α is inertia and α (r) is drag. The $\tau_u E$, $\tau_v E$ and $\tau_r E$ represent external disturbances and modelled dynamics of the system.

The actuator allocation matrix gives relation between the forces exerted by thrusters τ_1, τ_2, τ_3 and τ_4 and the forces that act on the rigid body (X, Y, N). For the case of the platform, whose actuator configuration is given in fig and $\theta = 45^o$, the actuator matrix is given as

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} \begin{vmatrix} \cos 45^o & \cos 45^o & \cos 45^o & \cos 45^o \\ \sin 45^o & -\sin 45^o & -\sin 45^o & \sin 45^o \\ D & -D & D & -D \end{vmatrix} \begin{vmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \,\&\, \tau_4 \end{vmatrix}$$

Since four actuators are allowed used to control three degrees of freedom (DOF), this presents an over actuated system. This allows for the design of fault tolerant control algorithms. The inverse actuator allocation matrix cannot be found, but a pseudo inverse can be calculated instead.

The actuators can be simply modelled using an affine model given as

$\tau_i = Kt|n_i|n_i$

Where n_i represents the individual thruster's control input.

Kt is the thruster coefficient and τ_i is thrust exerted by the i^{th} thruster.
The non-linear static thruster characteristic can easily be compensated within the control algorithm.

2.2. PATH FOLLOWING: VIRTUAL TARGET ALGORITHM

Kinematic model is represented as

$X = U \cos \varphi$
$Y = U \sin \varphi$
$\Phi = r$

Where

$U = \text{sqrt}\,(X^2 + Y^2)$

$\varphi = \text{arctg}\,(Y^2/X^2)$

With pre-defined Earth fixed frame {E} and body fixed coordinate system {B}, a Serret – Frenet frame {F} = [s y 0] is defined. Virtual target attached to {F} moves along pre defined path.

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Kinematic error model is obtained and expressed with respect to the frame {F}

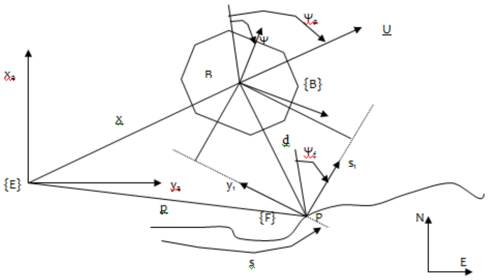

Fig. 3: Vehicle Parameter and Frame Definition

$s_1 = -s(1 - cy_1) + U \cos \beta$
$y_1 = - css_1 + U \sin \beta$
$\beta = r - cs$

Where s is speed of virtual target, attached to the frame {F}, c is the path curvature, U is vehicle's total velocity and β is difference between vehicle's direction of motion and orientation of virtual target which is the same as local path tangent at point P.

After some transient time, platform must be in origin of {F} frame, which means that S_1 and Y_1 converge to zero. Second task is to reduce β towards zero. When these requirements are fulfilled, path following is achieved. In order to design control law which ensure converges both the vehicle and virtual target, one of the possible Lyapunov function is chosen
$V = ½ (S_1^2 + Y_1^2)$

Computing its time derivative and after substitution are made, with proper choice of S, V_1 becomes negative-definite.

$S = U \cos \beta + K_2S_1$

As control signal which guarantees that $V_1 <$ 0, a new additional degree of freedom is introduced in the control structure. The speed of virtual target later takes a part in defining yaw rate reference. Considering the Lyapunov function

$V = ½ (\beta - \varphi)^2$

The above kinematic control law applies to the kinematic model of marine vehicles only. Using the back stepping techniques, this control law can be extended to deal with the vehicle dynamics. However, after that yaw rate reference is determined, a classic I-P controller is designed for yaw rate control. Back stepping technique supposes that process parameters are very accurately determined. If we have some uncertainty in mathematical model control signal can experienced chattering which is not acceptable behaviour. The high level control is based on the virtual target algorithm.

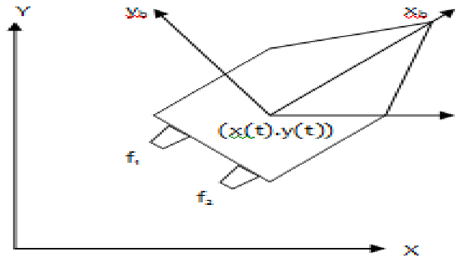

Fig. 4: Planar Model of A Surface Vessel With Two Propellers

The sliding mode controller for this under actuated unmanned surface vehicle is developed following the procedure. In general a smooth desired trajectory can be described by x = f (t), which is basically the position as a function of time. Then taking the time derivatives of the equation for finding the velocity and higher time derivatives as other functions of time is straight forward.

In the sliding mode control approach, we define asymptotically stable surface (S) such that all system trajectories converge to these surfaces in finite time referred to as the reach time, t_r and slide along them until they reach the desired destination at their intersection. The reaching conditions are normally established by defining $½S_TS$ as the Lyapunov function and ensuring that its time derivative is negative. In the case of under actuated surface vessels, we define two surfaces to determine the two control inputs. Hence, the reaching condition for each surface I may be defined as

$S_i\dot{S}_i <= - \eta_i|S_i|, \eta_i>0$, I = 1, 2, 3

Where the value of constant η_i determines how fast the trajectory will reach surface i;

$t_r <= S_i|/\eta_i|$

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2.3. SURGE CONTROL LAW

The first sliding surface is a first order exponentially stable surface defined in terms of the vessel's surge motion tracking errors.

$S_1 = \tilde{V}_\chi + \lambda_1 \int_0^t \tilde{V}^\chi (\tau) d\tau,\ \lambda_1 > 0$

Where "~" is used to denote the difference between the actual and desired values

$\tilde{V}_\chi = V_\chi - V_{\chi d}$

The integral of V_χ is used since position variables cannot be defined in the body fixed frame. Hence, the desired motion is specified in the inertial reference frame ($X_d(t)$, $Y_d(t)$) and its time derivatives (X_d, Y_d, $\dot{X}_d$, $\dot{Y}_d$) are related to the desired surge and sway velocities (V_{yd}, V_{xd}) and accelerations ($\dot{V}_{yd}$, $\dot{V}_{xd}$)

$V_{xd} = \cos\theta\, X_d + \sin\theta\, Y_d$
$V_{yd} = -\sin\theta\, X_d + \cos\theta\, Y_d$

$V_{xd} = \cos\theta\, \dot{X}_d + \sin\theta\, \dot{Y}_d + V_{yd}\omega$
$V_{yd} = -\sin\theta\, \dot{X}_d + \cos\theta\, \dot{Y}_d - V_{xd}\omega$

Θ is the actual measured yaw angle provided through feedback in real time and ω is estimated in real-time based on the θ values. Calculate nominal surge control law for zero dynamics by taking the time derivative of the surface and using the equation of motion

$\dot{S}_1 = \dot{V}_x + \dot{S}_{r1} = 0,\ \dot{S}_{r1} = -\dot{V}_{xd} + \lambda_1 \tilde{V}_x$
$\hat{f} = -\hat{m}_{22} V_y \omega + \hat{d}_1 V x^{\alpha 1} - \hat{m}_{11} \dot{S}_{r1}$

Where "^" is used to indicate the estimated model parameters. The sliding mode control law is normally derived by subtracting a sign function from the nominal control.

$f = \hat{f} - K_1 \mathrm{sat}(S_1/\phi_1)$

$$\mathrm{sat}(S_1/\phi_1) = \begin{cases} S_1/\phi_1 & \text{if } |S_1| \le \phi_1 \\ \mathrm{sgn}(S_1) & \text{if } |S_1| > \phi_1 \end{cases}$$

Where ϕ_1 is a positive constant, which defines a small boundary layer around the surface.
In order to determine K_1, defining the following bounds for the model parameters as

$|m_{ii} - \hat{m}_{ii}| \le M_{ii},\ |d_1 - \hat{d}_1| \le D_i,\ i = 1,2,3$

Assuming no uncertainty in our estimates of exponents α_1 for simplicity.
Define a Lyapunov candidate function that guarantees reaching the set $\{|S_1| \le \phi_1\}$ in finite time and remain inside it thereafter

$V_1 = ½\, m_{11} S_1^2$

The time derivative

$\dot{V}_1 = m_{11} S_1 \dot{S}_1$
$= m_{11} S_1 [(f + m_{22} v_y \omega - d_1 v_x^{\alpha 1})/m_{11} + \dot{S}_{r1}]$
$= S_1[(m_{11} - \hat{m}_{22}) V_y \omega + (\hat{d}_i - d_1) V_x^{\alpha 1} + (m_{11} - \hat{m}_{22}) \dot{S}_{r1} - k_1 \mathrm{sat}(S_1/\phi_1)]$

The saturation function is equal to the sign function of the set $\{|S_1| > \phi_1\}$. Hence, the following reaching condition is achieved as

$V_1 = m_{11} S_1 \dot{S}_1 \le -\hat{m}_{11} \eta_i |S_i|$

If k_1 is selected as

$k_1 = m_{22} |v_y \omega| + D_1 v_\chi^{\alpha 1} + M_{11} |\dot{S}_1| + \hat{m}_{11} \eta_i$

3. SYSTEM IDENTIFICATION AND MODELLING

System identification techniques have been applied to obtain the USV model and hence controllers are developed subsequently. It consists of four steps –

1. Data acquisition
2. Characterization
3. Identification
4. Verification

The first and most important step is to acquire the input/output data of the system to be identified. Acquiring data is not trivial and can be very laborious and expensive. This involves careful planning of the inputs to be applied so that sufficient information about the system dynamics is obtained. If the inputs are not well designed, then it could lead to insufficient or even useless data.

The second step defines the structure of the system, for example type and order of the differential equation relating the input to the output. This means the selection of a suitable model structure.

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The third step is identification, which involves determining the numerical values of the structural parameters, which minimize the error between the system to be identified and its model. Common estimation methods are least squares, instrumental variable, maximum likelihood and the prediction error method.

Where n_1 and n_2 are the two thrust propellers in revolutions per minutes. Straight line manoeuvres require both the thrusters to run at the same speed whereas the differential thrust is zero in this case. In order to linearism the model at an operating point, it is assumed that the vehicle is running at a constant speed of 3 knots. This corresponds to both thrusters running at 900 rev/min. To clarify this further, let n_c and n_d represent the common mode and differential mode thruster velocities defined to

$$n_c = (n_1 + n_2)/2$$
$$n_d = (n_1 - n_2)/2$$

In order to maintain the velocity of the vessel n_c must remain constant at all times. The differential mode input, however oscillates about zero depending on the direction of the manoeuvre. For data acquisition, several inputs including a pseudo random binary sequence are applied to the thruster. The input in the differential mode thruster velocity n_d causes the vehicle to manoeuvre. The acquired data were processed and down sampled to 1 Hz since this frequency was deemed to be adequate for controller design.

System identification was then applied to the acquired data set and a dynamic model of the vehicle is obtained using a prediction error method as

$$y(z) = G_1(z)u_1 + G_2(z)u_2$$

Where G_1 and G_2 denote the discrete transfer functions from inputs u_1 and u_2 respectively and where y is the output of the system. In this case, only n_d has been manipulated and therefore acts as the sole input to the system. This alters both n_1 and n_2 whereas n_c is maintained to conserve the operating regime. Two models of second and fourth order were identified from the data, however a subsequent simulation study reveals that there is no significant advantage in using a more complex fourth order model. Hence, the second order model in state space form is selected for further analysis and controller design.

4. CONCLUSION

A new simplified concept dealing with ocean waves and safe planning trajectory of USV is introduced.

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7. BIOGRAPHIES

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