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On Finding Integer Solutions to Sextic Equation With Three Unknowns $x^2+y^2=64z^6$

Assistant Professor Shanthi ,Prof. M.A.Gopalan
Department of Mathematics,
Shrimati Indira Gandhi College, Affiliated to Bharathidasan University,
Trichy-620 002, Tamil Nadu, India.
shanthivishvaa@gmail.com, mayilgopalan@gmail.com

Abstract- This paper deals with the problem of finding non-zero distinct integer solutions to the non-homogeneous ternary sextic equation given by $x^2+y^2=64z^6$.

Key words- non-homogeneous sextic ,ternary sextic ,integer''

I.INTRODUCTION

It is well-known that a diophantine equation is an algebraic equation with integer coefficients Involving two or more unknowns such that the only solutions focused are integer solutions.No doubt that diophantine equations are rich in variety [1-4] .There is no universal method available to know whether a diophantine equation has a solution or finding all solutions if it exists .For equations with more than three variables and degree at least three, very little is known.It seems that much work has not been done in solving higher degree diophantine equations.While focusing the attention on solving sextic Diophantine equations with variables at least three ,the problems illustrated in [5-21] are observed. This paper focuses on finding integer solutions to the sextic equation with three unknowns $x^2+y^2=64z^6$.

II.METHOD OF ANALYSIS

The non-homogeneous Diophantine equation of degree six with three unknowns to be solved in integers is

$$x^2+y^2=64z^6 \qquad (1)$$

Different ways of determining non-zero distinct integer solutions to (1) are illustrated below:

Way:1

Introduction of the transformations

$$x=m(m^2+n^2),\, y=n(m^2+n^2) \qquad (2)$$

in (1) leads to

$$m^2+n^2=4z^2 \qquad (3)$$

which is in the form of well-known Pythagorean equation. Using the most cited solutions of the Pythagorean equation and performing a few calculations ,the following two sets of integer solutions to (1) are obtained:

Set 1:

$$x=64\,(r^2-s^2)\,(r^2+s^2)^2,\, y=128\,rs(r^2+s^2)^2,$$
$$z=2(r^2+s^2)$$

Set 2:

$$x=[(2r+1)^2-(2s+1)^2)]\,[(2r+1)^2+(2s+1)^2]^2,$$
$$y=2(2r+1)(2s+1)[(2r+1)^2+(2s+1)^2]^2,$$
$$z=2(r^2+s^2+r+s)+1$$

Note 1:

It is worth to observe that ,in addition to (2) , one may consider the transformations

$$x=m(m^2-3n^2),\, y=n(3m^2-n^2)$$

and following the above procedure ,two more sets of integer solutions to (1) are obtained.

Way : 2

Introduction of the transformations

$$x=4(p^2-q^2)\,,\, y=8pq$$

(4)

in (1) leads to

$$p^2 + q^2 = 2z^3 \quad (5)$$

Assume

$$z = a^2 + b^2 \quad (6)$$

Write 2 on the R.H.S. of (5) as

$$2 = (1+i)(1-i) \quad (7)$$

Using (6) & (7) in (5) and employing the method of factorization ,define $p + iq = (1+i)(a+ib)^3$

from which ,on equating the real and imaginary parts, one obtains

$$\left.\begin{array}{l} p = a^3 - 3ab^2 - 3a^2b + b^3, \\ q = a^3 - 3ab^2 + 3a^2b - b^3 \end{array}\right. \quad (8)$$

From (4) , one has

$$\left.\begin{array}{l} x = 16\,(a^3 - 3ab^2)\,(-3a^2b + b^3) \\ y = 8[(a^3 - 3ab^2)^2 - (3a^2b - b^3)^2] \end{array}\right\} \quad (9)$$

Thus (6) and (9) represent the non-zero distinct integer solutions to (1).

Note:2

One may also express 2 on the R.H.S. of (5) as below:

$$2 = \frac{(7+i)(7-i)}{25}, 2 = \frac{(1+7i)(1-7i)}{25}$$

The repetition of the above process exhibits two more distinct integer solutions to (1).

Way:3

The substitution

$$x = 8X, y = 8Y \quad (10)$$

in (1) leads to

$$X^2 + Y^2 = z^6 = (z^3)^2 \quad (11)$$

The above equation (11) is in the form of Pythagorean equation .Employing the most cited solutions of the Pythagorean equation and after performing some algebra ,it is seen that (1) is satisfied by

$$x = 8(r^2 - s^2)(r^2 + s^2)^2, y = 16\,rs\,(r^2 + s^2)^2,$$

$$z = (r^2 + s^2)$$

III.CONCLUSION

In this paper ,an attempt has been made to determine the non-zero distinct integer solutions to the non-homogeneous ternary sextic diophantine equation given in the title through employing transformations. The researchers in this area may search for other choices of transformations to obtain integer solutions to the ternary sextic diophantine equation under consideration.

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