

# Mathematical Modeling of Blood Flow

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**Abstract-**This article review various effects of autonomous vehicles on society, the economy, the environment, and technology . The development of autonomous vehicle technology is advancing rapidly and it has the potential to completely alter people mobility in the future owing to developments in machine learning, artificial intelligence, and sensors. The promises come with a lot of difficulties, and ethical issues. Moreover, autonomous vehicles have the potential to significantly minimize accidents brought on by human mistake, improving traffic safety. They present the possibility of increased mobility for the elderly and disabled as well as improved traffic flow and less congestion. However, issues like the division of responsibility in cases of accidents and the coexistence of autonomous and human-driven vehicles need for careful consideration. In addition, autonomous vehicles have the ability to modify land use in urban planning and lessen the need for parking spots, enabling communities to consider new urban design. They affect conventional automotive sectors, labor markets, and business models economically, posing both disruption and opportunity. While, environmental sustainability is a major concern because autonomous vehicles may improve air quality, reduce emissions, and encourage the usage of electric vehicles. However, there is a need to manage the energy consumption caused on by computational demands. Futhermore, the introduction of autonomous vehicles raises concerns regarding privacy, algorithmic ethics, and changes in transportation behavior from a social and ethical standpoint. Case studies illustrate practical applications while highlighting the lessons and difficulties encountered. So that, in the future, it is crucial to Implementing complete automation, integrating with smart cities, and building global cooperation and regulatory frameworks. The successful integration of autonomous vehicles depends on gaining the public's trust and addressing issues of equity. Autonomous cars represent a revolutionary journey with a variety of potential and obstacles for a safer, more effective, and sustainable transportation system.

**Keywords-** autonomous, vehicle, technology, infrastructure, economic, environment.

## I. INTRODUCTION

**Mathematical approaches to study the effect of blood flow in human body.**

**Area of research** -Mathematical Modelling

**Objective of the proposal-**

blood pressure. This occurs when the blood vessel became narrowed from its normal size. A system of nonlinear partial differential equations for blood flow and the cross-sectional area of the artery were obtained. Finite difference method was adopted to solve the equations numerically.

### Background/short description

“Human blood is the fluid circulated by the heart through the human vascular system. There are three cellular components of human blood: red blood cells, white blood cells, and platelets. Red blood cells transport oxygen to other cells of the body and packed with Hemoglobin (an iron-bearing protein) and shaped like plump disks with indented centers, red blood cells are produced in bone marrow and have a life span of about 120 days. White blood cells protect the body from infection, attacking and destroying foreign particles like dust, pollen, and viruses. Platelets defend the body against excessive blood loss and platelets flow freely in the blood in an inactive state; but when an injury is sustained, platelets become sticky to plug the injured area (bloodwork.com/hematocrit- spun.html)”.

## II.ROLE OF BLOOD

- “Transports:
  - Dissolved gases (e.g. oxygen, carbon dioxide),
  - Waste products of metabolism (e.g. water, urea),
  - Hormones,
  - Enzymes,
  - Plasma proteins (associated with defense, such as blood-clotting and anti-bodies),
  - Nutrients (such as glucose, amino acids, micro-nutrients (vitamins & minerals), fatty acids, glycerol),
  - Blood cells (incl. white blood cells 'leucocytes', and red blood cells 'erythrocytes')
- “Maintains Body Temperature
- Controls pH
 

The pH of blood must remain in the range 6.8 to 7.4, otherwise it begins to damage cells.
- Removes toxins from the body
 

The kidneys filter all of the blood in the body (approx. 8 pints), 36 times every 24 hours. Toxins removed from the blood by the kidneys leave the body in the urine. (Toxins also leave the body in the form of sweat).
- Regulation of Body Fluid Electrolytes
 

Excess salt is removed from the body in urine, which may contain around 10g salt per day (Such as in the cases of people on western diets containing more salt than the body requires)

Some authors in the area of blood flow feel that blood can be assumed to be Newtonian in nature especially in large blood vessels such as the aorta” (Liu et al. (2004)). “However, blood flow may be

affected by non-Newtonian behavior of blood in smaller arteries and pathologically altered configurations such as aneurysms and stenosis, where red blood cells and platelets can aggregate and thus change blood viscosity”. “The fact that blood exhibits non-Newtonian behavior was actually first recognized around the turn of century. From a bio fluid mechanics point of view, blood would not be expected to obey very simple, one parameter, and linearized law of viscosity as developed by Newton. Blood is non homogeneous, anisotropic ionic, composite fluid composed of a suspension of many asymmetric, relatively large, viscoelastic particles carried in a liquid that contains high molecular weight, asymmetric, ionic that behaves in a complicated way under shear-type loading. Therefore, blood exhibits non-Newtonian (nonlinear), time dependent (viscoelastic) deformation (flow) characteristics that can only be modelled by higher order constitutive equations, such as the Power-law paradigm”.

## III.VISCOSITY OF BLOOD

“Blood is neither homogeneous nor Newtonian. Plasma in isolation may be considered Newtonian with a viscosity of about 1.2 times that of water. For whole blood, we measure effective viscosity, and this is found to depend on shear rate. Human Blood Circulation Model The human blood circulation system is a complex nonlinear system, and to study it quantitatively, we carried out our analysis based on [33]. The human body can be idealized into 14 chambers as shown in **Figure 5**, in which the blocks represent different chambers and the gray triangles represent valves between chambers.

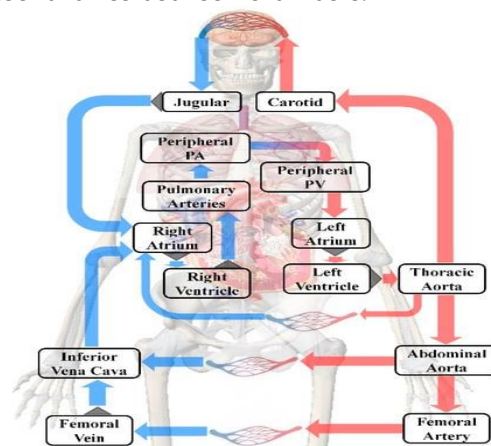


Figure 5. Schematic diagram of the human blood circulation model.

In the process of blood flow, the vessel wall expands and contracts with the change of pressure, which is the compliance of the vessel and can be quantified as:

$$C = \frac{\Delta V}{\Delta P} \quad (15)$$

where  $C$  is the compliance,  $\Delta V$  is the volume change, and  $\Delta P$  is the pressure change. During the flow of blood, the flow impedance and compliance given by the blood vessel are similar to the resistance and capacitance characteristics in a circuit, so vessels (Figure 6a) can be abstracted as circuits (Figure 6b).

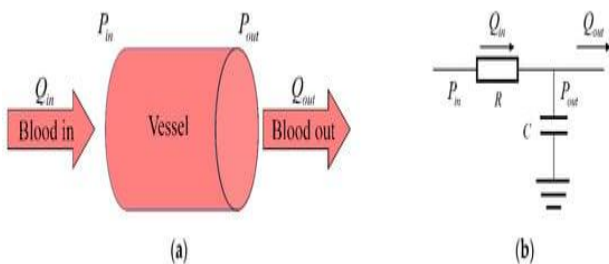


Figure 6. Vessel and its corresponding circuit: (a) vessel physiological model, (b) vessel corresponding circuit.

Where  $P_{in}$  and  $P_{out}$  are the blood pressure at both ends of the vessel, and  $Q_{in}$  and  $Q_{out}$  are the flow at both ends of the vessel.  $R$  and  $C$  are the resistance and compliance of the vessel, respectively. The blood flow  $Q_{in}$  can be calculated as:

$$Q_{in} = (P_{out} - P_{in}) / R \quad (16)$$

The blood pressure can be calculated as:

$$dP_{out} dt = (Q_{in} - Q_{out}) / C$$

#### IV. EXPECTED RESULTS

##### Flow Rate of Blood

"Flow through a blood vessel is determined by two factors: the force that pushes the blood through the vessel, and the resistance of the vessel to the blood flow. Ordinarily, the rate of blood flow is measured in millilitres or litres per minute (ml/min or l/min). The blood flow in the entire human circulation is about 5000 ml/min at rest in an average sized adult, but may be 5-6 times as great during heavy exercise when the body needs more oxygen to fuel that exercise. The amount of blood pumped by the heart in one minute is called the cardiac output. It is important to note that the flow of blood in the body is directly influenced by gravity.

When a person is standing, gravity helps pull the blood downward to the lower extremities. Without gravity, blood tends to remain closer to the heart. The force of gravity also makes it more difficult for the blood to flow upward to return to the heart and lungs for more oxygen. Our bodies have evolved to deal with the ever-present downward force of gravity; our leg muscles function as secondary pumps to help in the process of venous return which is blood flow back to the heart, also referred to as cardiac input. During walking or other leg movements, the muscles contract, forcing blood up through the veins of the calf toward the heart. The valves in the veins are arranged so that blood flows only in one direction. This mechanism effectively counteracts the force of gravity .

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