

Free Vibration Analysis of a Four Bar Mechanism

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Abstract- For obtaining high speed operation and improved efficiency, weights of many components in industrial robots and several machines are reduced. When a robot or a machine operates at high speeds, significant accelerations and dynamic forces may be produced. These dynamic forces may cause large dynamic deformations in one or more flexible components, system operational difficulties, premature failure of the components and so on. This project includes the analysis of mechanism for finding the displacement, velocity, acceleration, strain, stress and natural frequency. The numerical simulation by using general-purpose finite element software ANSYS is performed for Crank-Rocker elastic linkage to find natural frequencies and displacements in coupler and follower throughout the crank rotation.

Keywords:- ANSYS, Stress, Strain and FEM etc.

I. INTRODUCTION

The high speed mechanism's functioning introduces vibrations, acoustical radiations, joint detritions and incorrect positioning due to elastic connections deformations. For this it is necessary to perform an elasto-dynamic analysis of this type of problems rather than rigid body dynamic analysis.

The flexible mechanisms are flexible dynamic systems with infinite degrees of freedom and the motion equations are modeled as nonlinear partial differential equations. But their analytical solutions are difficult to achieve.

Dynamic equations of an FMS may be solved directly using numerical or analytical methods. Because the dynamical equations are usually very large in dimensions for a typical FMS, hence computing time for an accurate dynamic analysis may become a concern.

In this project, each flexible link is considered as a substructure and modeled as an elastic beam experiencing axial and lateral deformations. Three node higher order beam elements are employed to establish equations of motion for an unconstrained link. Under go considerable deformation. Omitting consideration of link deformations under dynamic conditions may contribute to the system operational

difficulties, component's premature failure and machine's failure to perform accurately and so on. Modeling and analysis of dynamic behavior of a flexible multi body system have attracted significant attention in the past few decades from many researchers. The finite element method can be applied easily to formulate component equations for an unconstrained individual link of complex geometry.

The Lagrange equations may then be used to establish dynamic equations of motion of a constrained Flexible multi body system. For small deformations, the equations of motion are a system of coupled second order ordinary differential equations with configuration dependent mass, stiffness and gyroscopic matrices.

II. LITERATURE SURVEY

The Lagrange equations may then be used to establish the system dynamic equations of motion of a constrained FMS. However, constraints between links are more complicated because multiple nodal displacements are involved in a set of holonomic constraint equations, which are dependent on time or system rigid-body configuration. The Lagrange multiplier method or the augmented. Lagrange equations method may be used to formulate the equations of motion satisfying all dynamic constraint

equations for small deformations, the equations of motion are a system of coupled second-order ordinary differential equations with configuration-dependent mass, stiffness and gyroscopic matrices.

Dynamic equations of an FMS may be solved directly using numerical or analytical methods. The holonomic constraints modeling the interactions between a flexible link and the base are easy to deal with because each constraint equation involves only a single nodal displacement.

III. METHODOLOGY FOR ANALYSIS OF FLEXIBLE FOUR BAR CRANK-ROCKER MECHANISM

1. An Unconstrained Flexible Link:

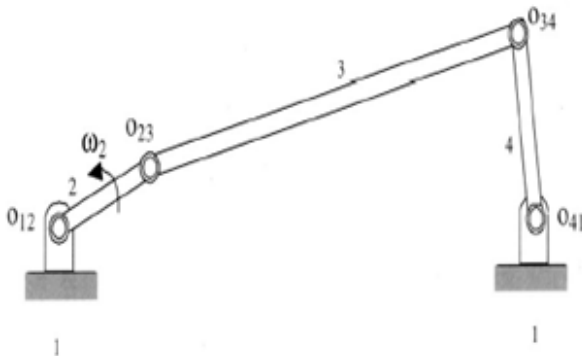


Fig 1. Rigid body configuration of a planar four-bar crank-rocker mechanism.

Elastic deflections of a flexible link vary with time and mechanism configuration, a set of moving coordinates fixed to each of the three moving links in a four-bar mechanism are employed.

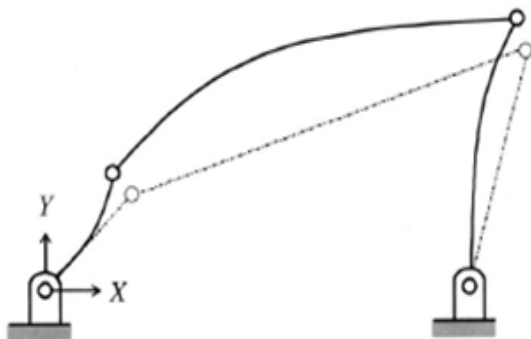


Fig 2. Deformed versus rigid body configurations.

The actual position of a material point in a flexible link is the simultaneous superposition of the elastic deflection caused by its own flexibility and flexibility

of all other links in the system. Point P, located at distance from the reference point on the neutral axis of flexible link i before deformations, as shown in Fig.3, moves to P' after deformations of all flexible links.

If the displacements measured with respect to the body-fixed coordinate system (x_iy_i) are u_i(x_i,t) in the longitudinal direction and v_i(x_i,t) in the lateral direction, the position of P' may be written as

$$\mathbf{R}_i = \mathbf{R}_{o,i} + \{ \mathbf{e}_{x,i} \quad \mathbf{e}_{y,i} \} \begin{Bmatrix} x_i + u_i \\ v_i \end{Bmatrix}$$

Where; $\mathbf{R}_{o,i}$ is the rigid-body position vector of reference point o_i; $\mathbf{e}_{x,i}$ and $\mathbf{e}_{y,i}$ are the two unit vectors of the body-fixed coordinate system. The velocity of point P_i may be written in the body-fixed coordinate system as

$$\dot{\mathbf{R}}_i = \begin{Bmatrix} v_{o,i,x} \\ v_{o,i,y} \end{Bmatrix} + \begin{Bmatrix} -\omega_i v_i \\ \omega_i (x_i + u_i) \end{Bmatrix} + \begin{Bmatrix} \dot{u}_i \\ \dot{v}_i \end{Bmatrix}$$

2. Lagrangian Method for Flexible Four-Bar Mechanism:

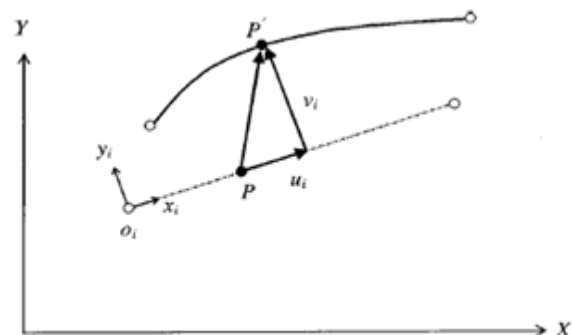


Fig 3. Deformed link and body-fixed coordinates.

The work potential associated with distributed axial force and flexural deformation may be determined from

$$W_i = \frac{1}{2} \int_0^{l_i} P_i \left(\frac{\partial v_i}{\partial x_i} \right)^2 dx_i = \frac{1}{2} \sum_{e=1}^{N_{e,i}} \{ q_i^e \}^T [K_{d,e,i}] \{ q_i^e \} \omega_i^2$$

where the dynamic stiffness matrix $[K_{d,e,i}]$ normalized with respect to the angular speed of the input link.

To derive the lagrangian for an unconstrained flexible link, the following set of link nodal variables for a total of NN nodes is introduced.

$$\{q_i\} = \{\bar{u}_1 \bar{v}_1 \quad \bar{\theta}_1 \quad \bar{u}_2 \quad \bar{v}_2 \quad \bar{\theta}_2 \cdots \bar{u}_{NN} \quad \bar{v}_{NN} \quad \bar{\theta}_{NN}\}_I$$

$\begin{matrix} \leftarrow J=0 & \leftarrow \text{nodes} & \leftarrow e=1 \end{matrix}$

IV. FINITE ELEMENT ANALYSIS AND ANSYS

1. Definition of FEA:

It can analyze elastic deformation, or "permanently bent out of shape" plastic deformation. The Computer is required because of the astronomical number of calculations needed to analyze a large structure. The power and low cost of modern computers has made Finite Element Analysis available to many disciplines and companies.

The term "Finite Element" distinguishes the technique from the use of infinitesimal "differential elements" used in calculus, differential equations, and partial differential equations. The method is also distinguished from finite difference equations, of which although the steps into which space is divided are finite in size, there is little freedom in the shapes that the discrete steps can take.

Finite element analysis is a way to deal with structures that are more complex that can be dealt with analytically using partial differential equations, FEA deals with complex boundary better than finite difference equations will, and give answers to real world structural problems. It has been substantially extended in scope during the roughly 40 years of its use.

2. Uses of FEA:

A considerable factor of ignorance can remain as to whether the structure will be adequate for all design loads. Significant changes in designs involve risk. Designs will require prototypes to be built and field-tested.

The field tests may involve expensive strain gauging to evaluate strength and deformation. With FEA, the weight of a design can be minimized, and there can be a reduction in the number of prototypes built. Field-testing will be used to establish loading on structures, which can be used to do future design improvements via FEA.

3. FEA as integral part:

By robustness of the finite element methods we mean that the performance of the finite element

procedures should not be unduly sensitive to the material data, the boundary conditions, and the loading conditions used. Therefore FE procedures that are not robust will also not be reliable.

For use in engineering design, it is of utmost importance that the finite element methods are reliable, robust and of course efficient. Reliability and robustness are important because a designer has relatively little time for the purpose of analysis and must be able to obtain an accurate.

4. ANSYS program:

The ANSYS computer program is a large-scale multipurpose finite element program that may be used for solving several classes of engineering analyses. The analysis capabilities of ANSYS include the ability to solve static, dynamic, steady-state, transient heat transfer problems, mode frequency, buckling eigen value problems, static or time varying magnetic analysis, various types of coupled field applications.

The program contains many special features, which allow non-linearities or secondary effects to be included in the solution, such as plasticity, large strain, hyper elasticity, creep, swelling, large deflections, contact, stress stiffening, temperature dependency, material anisotropy and radiation.

5. Procedure for ANSYS analysis:

5.1 Modeling with ANSYS: The modeling procedure is the following:

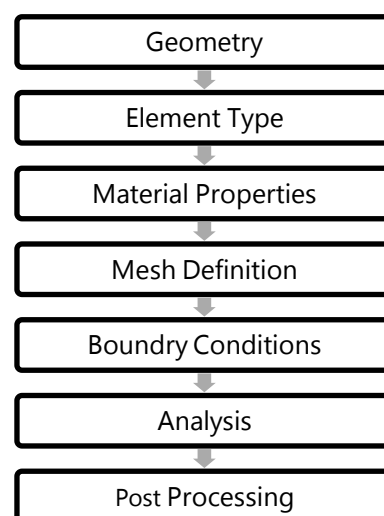


Fig 4. Modelling Procedure.

V. FINITE ELEMENT ANALYSIS OF FOUR BAR FLEXIBLE MECHANISMS

1. Kinetics of Four-Bar Mechanism:

In this section, kinetic analysis of an example for four-bar mechanism is presented for rigid and flexible models. Flexible four-bar mechanism is modeled by using beam finite elements in ANSYS. The following subsections give the details for the solution procedure developed in this study.

2. Vibrations of the Flexible Mechanism:

For the mechanism of which parameters are given in previous sections, the natural frequencies and corresponding mode shapes are presented.

The numerical simulations were performed for the crank-rocker flexible mechanism by using NX Software to find the co-ordinates of the link positions. The geometric and material properties for the mechanism are presented in Table 1.

The eigen analysis of a flexible four-bar mechanism as an instantaneous structure without the consideration of gyroscopic effects and motion induced axial load is performed by using general purpose finite element analysis software ANSYS.

Table 1. Geometric and material properties of Crank-Rocker flexible mechanism.

Links	L mm	b mm	h mm	E Gpa	ρ kg/m ³
Ground	254.0	4.24 1.6 1.6	25.4 25.4 25.4	68.9 68.9 68.9	2698 2923 2923
Input	108.0				
Coupler	279.4				
Follower	270.5				

3. Analysis of mechanism when it is under motion using Transient Analysis:

Only planar motion is assumed and small elastic deformations from the rigid body equilibrium positions are considered.

The gravitational acceleration is assumed small compared to the rigid body and elastic accelerations. In the results that are given only the first three modes are considered. The crank is given a constant counter clock wise angular velocity of 35.60 rad/sec (340 rpm).

Each link is idealized as four beam elements. The assumptions made are bearings with-out friction and without play.

VI. RESULTS

1. Analysis of mechanism as an instantaneous structure at different positions:

The first five natural frequencies of the flexible mechanism at two configurations defined by $\theta_2 = 0^\circ$ and $\theta_2 = 90^\circ$.

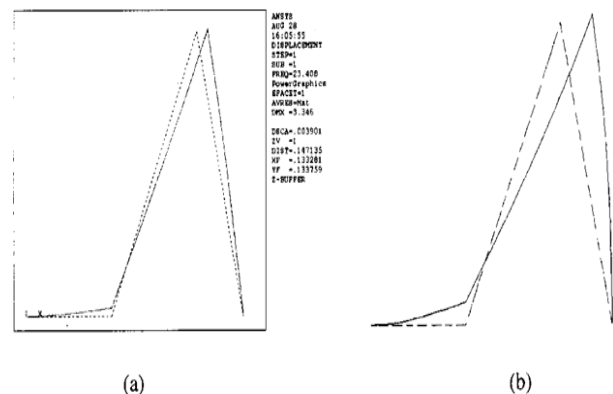


Fig 5. The 1st structural vibration mode shape of the first example mechanism at $\theta_2 = 0^\circ$

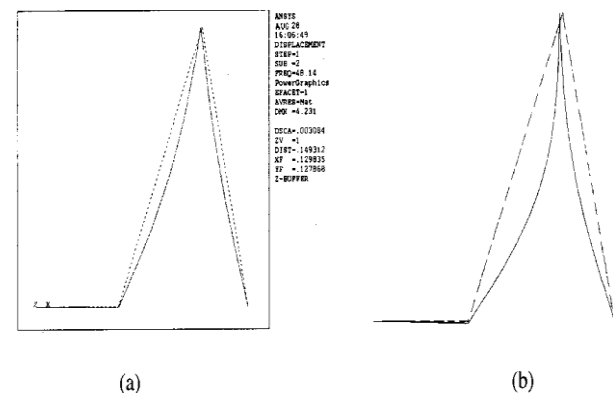


Fig 6. The 2nd structural vibration mode shape of the first example mechanism at $\theta_2 = 0^\circ$

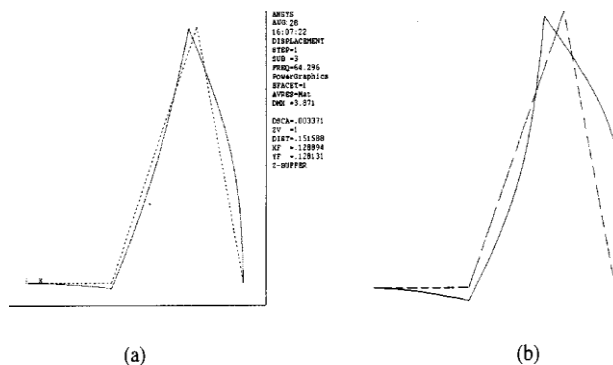


Fig 7. The 3rd structural vibration mode shape of the first example mechanism at $\theta_2 = 0^\circ$

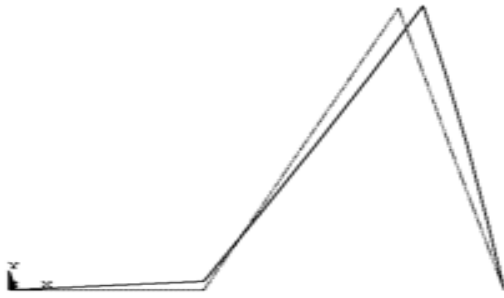


Fig 8. First mode shape of flexible four-bar mechanism for $\theta_2=0^\circ$.

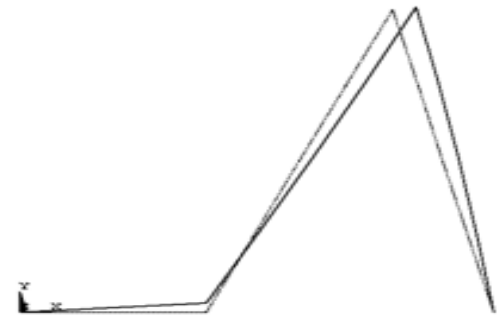


Fig 9. Second mode shape of flexible four-bar mechanism for $\theta_2=0^\circ$



Fig 10. Third mode shape of flexible four-bar mechanism for $\theta_2=0^\circ$.

2. Natural Frequencies of Mechanism with Internal Force:

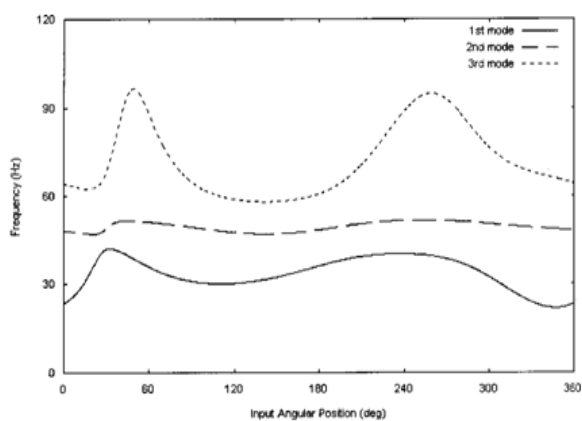


Fig 11. Variation of the first three natural frequencies of the first example mechanism throughout a cycle.

Inertia forces acting on the lumped masses of the mechanism are considered in finite element model created in ANSYS. The internal force due to the inertia force is taken into account in finding the natural frequencies of mechanism for different angular velocities.

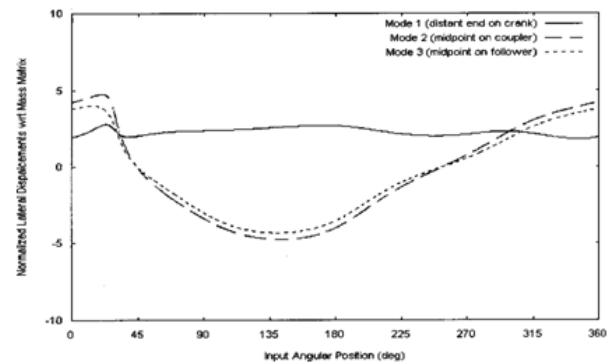


Fig 12. Variation of most prominent lateral displacements in the first three modal vectors of the first example mechanism throughout a cycle.

3. Results for Analysis of mechanism when it is under motion using Transient Analysis:

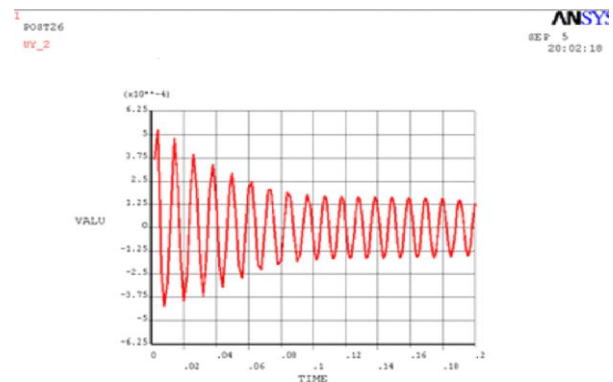


Fig 13. Distribution of Displacement of Coupler mid point Versus Crank rotation.

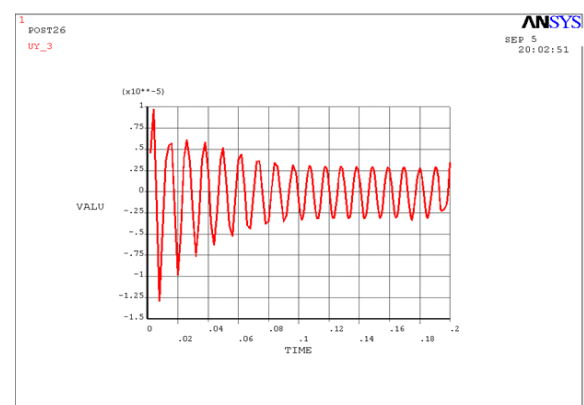


Fig 14. Distribution of Displacement of Follower mid point Versus Crank rotation.

VII. CONCLUSION

This paper presents a free vibration analysis of a Crank-Rocker flexible planar mechanism using a combination of finite element method. This work concerns the development of general approach to the formulation of equations of motion for complex elastic mechanism systems. Lagrange multipliers method. This work may be extended to find the dynamic natural frequencies of the mechanism while it is moving with high speed.

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