Niranjan Panda, 2021, 9:3
ISSN (Online): 2348-4098
ISSN (Print): 2395-4752

Differential Equations

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Abstract- In this study, I discuss different types of differential equations i.e. ordinary differential equations, partial differential equations with order and degree. Also defined linear differential equation and Bernoulli's equation.

Keywords: - Differential equation, PDE, Linear differential equation, Bernoulli's differential equation.

I. INTRODUCTION

As in Engineering and Technology, social sciences, the general laws of medicine, population dynamics and everywhere all the models or mathematical models are in the form of differential equations.

So differential equations are very essential topic for every student. Here, in this paper discussed methods of solution of first order and degree ordinary differential equations.

II. DIFFERENTIAL EQUATION

If the equation involves by differential terms then it said to be differential equation.

We can write $\frac{dy}{dx} = f(x)$, here "x" is an independent variable and "y" is a dependent variable, for example, $\frac{dy}{dx} = 10x$.

Differential Equations are categorized into Ordinary differential equation and Partial differential equations.

1. Ordinary Differential Equation:
If the differential equation containing one independent variable then it is said to be an ordinary differential equation.

Example:

(1) $\frac{d^2y}{dx^2} + k^2y = 0$

(2) $\left(\frac{dy}{dx}\right)^3 + \left(\frac{dy}{dx}\right)^2 - 10y = \sin y$

The general solution of an ordinary differential equation is

$$F\left(x, y, \frac{dy}{dx}, \frac{d^2y}{dx^2}, \dots, \frac{d^ny}{dx^n}\right) = 0$$

2. Partial Differential Equation:
If the differential equation containing more than one independent variable then it is said to be a Partial differential equation (PDE).

Example:

(1) $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

(2) $\frac{\partial^2 u}{\partial x \partial y} = 0$

3. Order and Degree of a Differential equation:

- The order of a differential equation is the order of the highest derivative terms of the differential equation.
- The degree of a differential equation is the power of the highest differential terms of the differential equation.

III. EQUATIONS

If the equation is of the form $\frac{dy}{dx} = f(x,\ y)$, then it is said to be a differential equation of first order and first degree. For examples linear differential equation and Bernoulli's differential equation.

1. Linear Differential Equations of First Order:
There are two types of Linear differential equations form that is

$\boldsymbol{\frac{dy}{dx} + P(x)y = Q(x)}$with respect to 'x' and

$\boldsymbol{\frac{dx}{dy} + P(y)x = Q(y)}$with respect to 'y'

Where; P and Q are constants.

To solve first kind of the linear differential equation i.e. $\frac{dy}{dx}+P(x)y=Q(x)$.

- First write the differential equation in the form $\frac{dy}{dx}+P(x)y=Q(x)$.
- Find $P(x)$ and $Q(x)$.
- Find the integrating factor $I.F. = e^{\int P(x)\,dx}$.
- The general solution is
  $$y\times(I.F.) = \int Q(x)\times(I.F.)\,dx + c.$$

To solve second kind of the linear differential equation i.e. $\frac{dx}{dy}+P(y)x=Q(y)$.

- First write the differential equation in the form $\frac{dx}{dy}+P(y)x=Q(y)$.
- Find $P(y)$ and $Q(y)$.
- Find the integrating factor $I.F. = e^{\int P(y)\,dy}$.
- The general solution is
  $$x\times(I.F.) = \int Q(y)\times(I.F.)\,dy + d.$$

2. Bernoulli's Differential Equation:

If the differential equation is of the form $\frac{dy}{dx}+P(x)y=Q(x)y^n$, then it is said to be Bernoulli's equation.

To solve Bernoulli's differential equation

- Multiply y^{-n} both sides of the above equation, then above equation becomes
  $$\frac{dy}{dx}y^{-n}+P(x)y^{1-n}=Q(x)\ldots\ldots\ldots\ldots\ldots(1)$$
- Let $y^{1-n}=v$ so that $(1-n)y^{-n}\frac{dy}{dx}=\frac{dv}{dx}$
  $$i.e.\ \ y^{-n}\frac{dy}{dx}=\frac{1}{(1-n)}\frac{dv}{dx}\ldots\ldots\ldots\ldots(2)$$
- From equation (1) and (2), we get
  $$\frac{1}{(1-n)}\frac{dv}{dx}+P(x)v=Q(x)$$
  $$i.e.\ \ \frac{dv}{dx}+(1-n)Pv=(1-n)Q.$$

Which is a linear differential equation of first order in *v* and this can be solved by using linear differential equation method. To get general solution put the value of *v*.

IV. CONCLUSION

In this paper, I have discussed ordinary differential equation, order and degree with proper examples. Also defined the procedures to solve the Linear differential equations and Bernoulli's differential equation.

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