Olaleye Olalekan A, *et al.* 2021, 9:4
ISSN (Online): 2348-4098
ISSN (Print): 2395-4752

Mathematical Modelling of Effects of Thermal Conductivity of Some Substances on Their Temperature in the Presence of Magnetic Field

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Abstract- The effect of thermal conductivity of various substances on their temperature in the presence of magnetic field was modelled mathematically. The equation was first modelled in their dimensional forms resulting into second order partial differential equation but later reduced to non-dimensional form using some standard dimensionless parameters which were obtained from some existing literatures. This energy governing equation was then solved analytically after it has been converted to ordinary differential equation with the aid of perturbation method. Adopting some default parameters, the numerical computations were made by the use of Matlab R2009b. The effects of various thermal conductivities on the temperature of some substances were then examined. Moreover, the effects of the Prandtl number and time were also considered on the temperature of loamy soil. It was clear according to the results that as the thermal conductivity increased, so the temperature of these substances was boosted.

Keywords- Dimensionless parameters, magnetic field, Prandtl number, thermal conductivity.

I. INTRODUCTION

The study of heat transfer in many fields is a very crucial one. This has drawn attention of many scientists from physics, Astrology, mechanical engineering, civil engineering, building technology, and so on. Heat is known to flow from a substance with higher temperature to that with a lower temperature when they are in contact with each other.

Moreover, when an edge of a substance that possesses ability to conduct heat comes in contact with a heat source, the heat is transferred from within the same substance from the edge that makes a contact with the source of heat to the cooler part. These principles that guided the transfer are however very useful in different fields of engineering and technology. Heat transfer is also applicable in agriculture which involves soil temperature suitable different crops. Soil temperature, an indication of the degree of hotness or coldness of a particular soil, Influences planting, seed germination, irrigation, tillage and other agricultural operations [1]. Soil temperature, influenced mainly by solar radiation, rainfall/ irrigation and soil physical properties, affects root development, seed germination and seedling emergence/growth. It also influences other processes in the soil- plant- air- water system and the timing of tillage, planting, irrigation and other agricultural activities. Soil temperature exhibits spatial and temporal variability [2], [3], [4], [5].

According to the Statement of Guidance (SOG) for Agricultural Meteorology [6], in addition to the standard weather elements such as air temperature, precipitation, relative humidity, wind speed/direction and solar radiation, it is important to also collect soil moisture and soil temperature data at strategically located stations. These data are critical for monitoring drought, satellite remote sensing ground truth procedures and soil moisture model initialization and verification. Optimum monitoring of soil moisture requires measurements to depths of 50-100 cm every 5-7 days. Soil temperature directly influences crop growth by providing necessary warmth to seeds, plant roots, and microorganisms within the soil profile. The physico-chemical as well as life processes are also directly affected by the temperature of the soil. Under low soil temperature conditions nitrification is inhibited and the intake of water by roots is reduced. Extreme soil temperatures injure plants affecting growth and development [6].

Moreover, the report of the study done by Harris, Frances and Mohammed [7] revealed that the main crops grown in the northern region of Nigeria are millet, sorghum, and cowpea while groundnut and sesame are significant minor crops. The main crops in this area are millet, sorghum, and cowpea, while groundnut and sesame are considered minor crops. Guna (cow) melon (Citrullus lanatus) has been recently added to crop production for market. It requires only one fall of rain, is planted late in the season, and grows on moisture that is left in the soil until it is harvested during the months preceding the next year's rains. This crop is a significant supply of edible oil [8]. Plants grown as field boundaries include the henna bush (Lawsonia inermis) as well as various grasses. Intercrop spreads are also planted among the grains; they often consist of cowpeas or groundnuts, which are nitrogen-fixing plants. A density of mature trees is also maintained [9].

In engineering, for example, in the processes such as drying, evaporation at the surface of water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its frequent occurrence in many branches of science and engineering, [10].

II.MATHEMATICAL ANALYSIS

The heat transfer flow was assumed to be three dimensional. Then, the horizontal axes (i.e. x and y) were decomposed to become one axis (y- axis) and later considered to be infinite. This automatically left the flow to be functions of the vertical axis (z – axis) and the time (being an unsteady flow). The radiation involved is from the solar source and the major substance considered is loamy soil which is an example of a porous medium that permits heat transfer. The flow is also assumed to be in the presence of magnetic field (such as the earth's magnetic field). With the above conditions, using the Boussinesq's approximation, the equations that govern the flow is given below:

Continuity Equation

$$\frac{\partial w^*}{\partial z^*} = 0 \tag{1}$$

Energy Equation

$$\frac{\partial T^*}{\partial t^*} + w^* \frac{\partial T^*}{\partial z^*} = \frac{1}{\rho C_p}\left\{\frac{\partial}{\partial z^*}\left(k\frac{\partial T^*}{\partial z^*}\right)\right\} - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial z^*} + \frac{\sigma B^2 w^2}{\rho C_p} \tag{2}$$

Subject to:

$$T^* = T_\infty^* + (T_w^* - T_\infty^*)\left[1 + A\cos\left(\frac{\pi t^*}{L}\right)\right] \text{at } z^* = 0 \tag{3}$$

$$T^* = T_\infty^* \quad \text{as} \quad z^* \to \infty \tag{4}$$

where z^* is the vertical axis which measures the soil's depth, w^* is suction velocity, ρ is density, t^* is time, C_p is specific heat capacity, T^* is temperature, q_r^* is radiative heat flux, k is thermal conductivity, T_w^* and T_∞^* are the wall and free stream temperatures respectively, B and σ are respectively the intensity of magnetic field and electrical conductivity.

In line with Mohammed [10], a constant suction velocity is adopted. This is set as:

$$w^* = -w_0(1 + \varepsilon\lambda e^{i\omega^* t^*}) \tag{5}$$

The soil is absorbing heat from electromagnetic waves released by the sun. Hence, follows Akinpelu *et al* [11], radiative heat flux is given as:

$$\frac{\partial q_r^*}{\partial z^*} = 4\alpha^2 (T^* - T_\infty^*) \tag{6}$$

Furthermore, relating to Akinpelu *et al* [12], thermal conductivity is considered to be changing with time and given as:

$$k = k_0(1 + \gamma t) \tag{7}$$

Bringing in the subsequent dimensionless parameters,

$$w = \frac{t^* w_0^2}{t}, z = \frac{w_0 z^*}{w}, \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \omega = \frac{w\omega^*}{w_0^2}, t = \frac{t^*}{L} \tag{8}$$

Substituting equations (5) – (8) into equation (2), this yields:

$$\frac{\partial\theta}{\partial t}-(1+\varepsilon\lambda e^{i\omega t})\frac{\partial\theta}{\partial z}=\left(\frac{1+\gamma t}{P_r}\right)\left(\frac{\partial^2\theta}{\partial z^2}\right)-R^2\theta+(Ha)^2Ec \tag{9}$$

where,

$$P_r=\frac{w\rho C_p}{k_0}\quad\text{(the Prandtl number)}$$

$$R=\frac{2\alpha}{w_0}\sqrt{\frac{w}{\rho C_p}}\quad\text{(the radiation parameter)}$$

$$Ha=B\sqrt{\frac{\sigma}{\rho}}\quad\text{(the Hartman number)}$$

$$Ec=\frac{w^3}{w_0^2C_p(T_w^*-T_\infty^*)}\quad\text{(the Eckert number)}$$

Also, substituting equation (8) into equation (3) and (4), the boundary conditions became:

$$\theta=1+A\cos(\pi t)\qquad at\qquad z=0 \tag{10}$$

$$\theta\to 0\qquad as\qquad z\to\infty \tag{11}$$

III.METHOD OF SOLUTION

Adopting the method of perturbation in order to reduce equation (10) to ordinary differential equation as used by Nwaigwe [13], the assumed solution is given as below:

$$\theta(z,t)=\theta_0(z)+\varepsilon e^{i\omega t}\theta_1(z)+o(\varepsilon^2)+\cdots \tag{12}$$

Differentiating equation (13) and substituting it into equation (10) alongside equation (13) itself, it yielded:

$$\frac{d^2\theta_0}{dz^2}+\frac{P_r}{1+\gamma t}\frac{d\theta_0}{dz}-\frac{P_rR}{1+\gamma t}\theta_0=\frac{-P_r(Ha)^2Ec}{1+\gamma t} \tag{13}$$

$$\frac{d^2\theta_1}{dz^2}+\frac{P_r}{1+\gamma t}\frac{d\theta_1}{dz}-\frac{P_r(i\omega+R)}{1+\gamma t}\theta_1=\frac{\lambda P_r}{1+\gamma t}\frac{d\theta_0}{dz} \tag{14}$$

Moreover, by rewriting the boundary conditions (10) and (11) with the use of equation (12), this also became;

$$\theta_0=1+A\cos(\pi t)\ ,\ \theta_1=0\ at\ z=0 \tag{15}$$

$$\theta_0\to 0,\ \theta_1\to 0\qquad as\qquad z\to\infty \tag{16}$$

Solving equations (13) and (14) subject to equations (15) and (16), the transient soil temperature simply became;

$$\theta=D_1e^{m_1z}+D_2e^{m_2z}+D_3+\varepsilon e^{i\omega t}(D_4e^{m_3z}+D_5e^{m_4z}+D_6e^{m_1z}+D_7e^{m_2z}) \tag{17}$$

where,

$$m_1=-\frac{P_r}{2(1+\gamma t)}+\sqrt{\frac{P_r^2}{4(1+\gamma t)^2}+\frac{RP_r}{1+\gamma t}}$$

$$m_2=-\left(\frac{P_r}{2(1+\gamma t)}+\sqrt{\frac{P_r^2}{4(1+\gamma t)^2}+\frac{RP_r}{1+\gamma t}}\right)$$

$$m_3=-\frac{P_r}{2(1+\gamma t)}+\sqrt{\frac{P_r^2}{4(1+\gamma t)^2}+\frac{P_r(i\omega+R)}{1+\gamma t}}$$

$$m_4=-\left(\frac{P_r}{2(1+\gamma t)}+\sqrt{\frac{P_r^2}{4(1+\gamma t)^2}+\frac{P_r(i\omega+R)}{1+\gamma t}}\right)$$

$$D_1=-\frac{D_3}{e^{m_1z}}$$

$$D_2=1+A\cos(\pi t)-D_1-D_3$$

$$D_3=\frac{(Ha)^2Ec}{R}$$

$$D_4=-\frac{D_6e^{m_1z}}{e^{m_3z}}$$

$$D_5=-D_4-D_6-D_7$$

$$D_6=\frac{\left(\frac{\lambda P_r}{1+\gamma t}\right)m_1D_1}{m_1^2+\left(\frac{P_r}{1+\gamma t}\right)m_1-\frac{P_r(i\omega+R)}{1+\gamma t}}$$

$$D_7=\frac{\left(\frac{\lambda P_r}{1+\gamma t}\right)m_2D_2}{m_2^2+\left(\frac{P_r}{1+\gamma t}\right)m_2-\frac{P_r(i\omega+R)}{1+\gamma t}}$$

IV.RESULTS AND DISCUSSION

One of the substances that was considered here is the Loamy soil which thermal conductivity according to Gary [14] is given to be 0.52 Btu/ft hr^{0}F. Then the thermal conductivity was raised which can be as a result of increasing moisture content at various degree of percentage. Moreover, different soils have different thermal conductivity. Such include unsaturated sandy soil with about 0.44 Btu/ft hr^0F [14] but is raised to 1.44 Btu/ft hr^0F [14] when the soil is saturated with a level of water content. Equation (17) being the solution to the model that was formed in non-dimensional form according to equation (9) was used in examining the effect of the thermal conductivities of these substances.

Also, adopting some default parameters in some existing works like that of Mohammed [10], Matlab software was used for the aspect of the numerical computations. These parameters were shown on table 1 below. The results were then displayed on graphs to better illustrate them. Moreover, the effect of the Prandtl number which higher number implies smaller values of thermal conductivities and its smaller figures means that the thermal conductivity has higher values, was also examined on the temperature of the substance. Furthermore, the temperature of the substance which thermal conductivity is 0.52 Btu/ft hr^0F, i.e. the Loamy soil (Gary, 2015) was observed as time passes but all other parameters were kept constant.

Table – 1: Default values of important parameters involved

P_r	R	Ha	Ec	γ
0.71	0.10	1.00	0.01	0.52
t	**A**	**ω**	**ε**	
0.10	1.00	$\pi/2$	0.01	

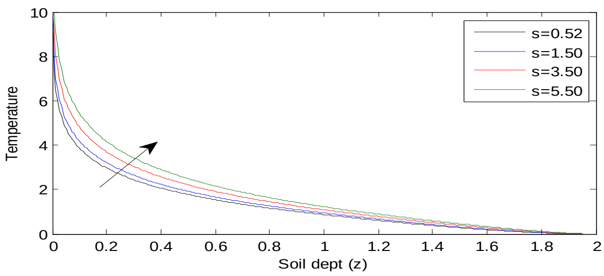

Figure 1: Effects of variation in thermal conductivity on the temperature of various substances.

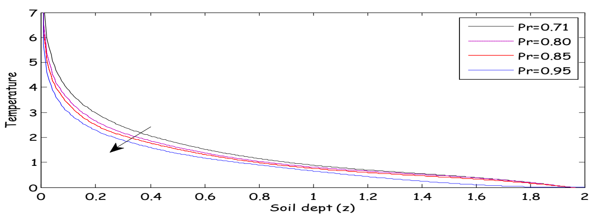

Figure 2: Effects of variation in Prandtl number on the temperature of loamy soil.

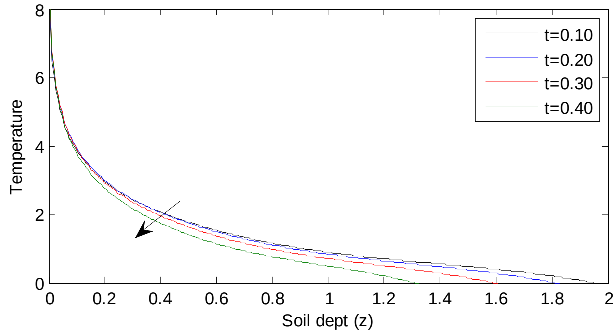

Figure 3: Effects of variation in time on loamy soil temperature.

In Figure 1, Effects of variation in thermal conductivity on the temperature of various substances was depicted. The loamy soil was first considered which has the thermal conductivity of 0.52 Btu/ft hr^0F, and then gradually increased it to 1.5 Btu/ft hr^0F, then to 3.5 Btu/ft hr^0F until it reached 5.5 Btu/ft hr^0F. It is seen that when the thermal conductivity was increases, there is also a rise in the temperature of the substance.

According to figure 2, which is the effect of variation in Prandtl number on the temperature, the loamy soil was considered also with the above thermal conductivity value. The Prandtl number was gradually increased which led to the decrease in the temperature of the substance. This is because a lager Prandtl number is tantamount to a lower value of the thermal conductivity, while a smaller Prandtl number is equivalent to a larger thermal conductivity. In figure 3, all other parameters are kept constant and as time passes, the temperature of the loamy soil was observed. It is observed that the temperature decreased at increasing depth of the loamy soil.

V.CONCLUSION

The study of the effects of thermal conductivity of various substances on their temperature in the presence of magnetic field was carried out. It was discovered that the thermal conductivity have significant effect on the temperature of the soil. A boost in this factor will enhance the temperature of the substance/soil involved.

REFERENCES

[1]. Ogunlela A. O. (2014). Spectral Analysis of Soil Temperature Data for Ilorin, Nigeria. Proceedings of the International Soil Tillage Research

Organisation (ISTRO) Nigeria Symposium, Akure 2014 November 3 - 6, Akure, Nigeria, 188 – 194.

[2]. Hilel, D. (1980). Fundamentals of Soil Physics. Academic Press.Inc.

[3]. Elias, E. A., Cichota, R., Torriani, H.H. and de Jong Van Lier Q. (2004). Analytical soil temperature model correction for temporal variation of daily amplitude. Soil Sci. Soc. Am. J. 68: 784- 788.

[4]. Ogunlela, A. O. (2003). Modelling soil temperature variations. J. Agric. Res>& Dev. 2: 100- 109.

[5]. Ogunlela, A. O. (2011). Confidence limits on soil temperature data. Proc. International Soil Tillage Research Organization (ISTRO – Nig): 270- 277.

[6]. Statement of Guidance for Agricultural Meteorology. March 2011 version approved by ET-EGOS-6, June 2011, significant revision of previous SOG

[7]. Harris, Frances M. A., and Salisu Mohammed (2003). "Relying on Nature: Wild Foods in Northern Nigeria." Ambio 32: 24-9.

[8]. Adams, William. M., and M. J. Mortimore (1997). Agricultural Intensification and Flexibility in the Nigerian Sahel. The Geographical Journal 163.2, Environmental Transformations in Developing Countries: 150-60.

[9]. Mortimore, Michael J. (2005). "Dryland Development: Success Stories from West Africa." Environment.45: 10-21.

[10]. Mohammed Ibrahim S. (2013). Radiation Effects on Mass Transfer Flow through a Highly Porous Medium with Heat Generation and Chemical Reaction, ISRN Computational Mathematics. Article ID 765408, 9 pages

[11]. Akinpelu F. O., Olaleye O. A. and Alabison R. M. (2020). A Comparison of the Effects of Solar Radiation on Sandy-Loam and Clay-Loam Soils with Convective Boundary Condition, International Annals of Science, 8(1): 130 – 137.

[12]. Akinpelu F. O., Alabison R.M. and Olaleye O. A. (2016). Variations in Ground Temperature in the Presence of Radiative Heat Flux and Spatial-Dependent Soil Thermophysical Property, International Journal of Statistics and Applied Mathematics , 2 (1), 57-63.

[13]. Nwaigwe, C. (2010). Mathematical Modeling of Ground Temperature with Suction Velocity and Radiation, American Journal of Scientific and Industrial Research, 238-241.

[14]. Gary Reysa (2015). Ground Temperatures as a Function of Location, Season and Depth, Build it Solar, The Renewable Energy Site for Do-It-Yourselfers. Sited at: www.buiolar.com/Projects/Cooling/EarthTemperatures.htm

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