Lee Pashouros, 2021, 9:6
ISSN (Online): 2348-4098
ISSN (Print): 2395-4752

The Effect of Scale on the Phenomenon of Classical Hydraulic Jump in Open Channels

Lee Pashouros
Department of Civil Engineering,
School of Architecture,
Design and the Built Environment, Nottingham
Trent University, Nottingham NG1 4FQ,
United Kingdom.

Roshan P De Silva
Department of Civil Engineering
School of Built Environment Engineering & Computing,
Leeds Beckett University,
Northern Terrace, Leeds LS2 8AG,
United Kingdom.

Corresponding author. Tel.: +44(0) 113 812 0000;
Email address: r.p.de-silva@leedsbeckett.ac.uk

Abstract- This study looked at the validity of accepted theories in relation to the classical hydraulic jump phenomenon when carried out at small scale. The investigations were performed using two small flumes and a total of 87 sets of data were recorded which included the depths of flow, energy loss and length of the hydraulic jump which were compared with the calculated values obtainable from the accepted theories which are used in real world situations with success. As seen from the results, calculated parameters agree quite well with measured counterparts of the same parameters, except possibly for the length of the hydraulic jump, which is possibly due to the empirical nature of the equations used for calculating the length of the jump and also due to the difficulty in identifying the jump precisely in an experimental setting due to the high turbulence and varying nature of the start and end of the jump in a flume. Though the results tend to agree more for the bigger of the two flumes better, it is concluded that with the scale that was used in the experiment, the effect of it on the phenomenon of hydraulic jump is very likely negligible.

Keywords- Effect of scale, Hydraulic jump, Energy head loss, Jump length.

I. INTRODUCTION

The classical hydraulic jump phenomenon (which will be referred to as the hydraulic jump from here on), is the transition of water flowing from high velocity, supercritical flow to low velocity, subcritical flow in rapid succession and has been studied in greater detail in the documented literature [1-4].

Hager [5] gives a detailed account of the historical contribution to the development of hydraulic jump theory. Though Leonardo da Vinci in the 1500s is thought to have studied the hydraulic jump, the first documented study on this phenomenon is attributed to Bidone in 1819 and to Belanger in 1841 [6, 7, 5].

Belanger is credited in general with analyzing the profile, the jump length and the velocity distribution of the jump.

This spectacle of the hydraulic jump occurs both naturally and artificially and Figure 1 below displays how a hydraulic jump is replicated in laboratory conditions, using a flume and a sluice gate which creates the initial super critical velocity required for the formation of the jump.

Fig 1. Formation of a hydraulic jump in laboratory conditions.

As discussed in Palermo and Pagliara, [7], Belanger's analysis, which was limited to smooth channel beds on prismatic canals, could not analyze other situations of the hydraulic jump with variable factors like rough beds, sloping channels, non-prismatic channels and the effect of having high upstream Froude numbers.

As such, many researchers conducted experimental as well as numerical studies to understand not only the hydraulic jump phenomenon, but also the complexities associated with it under different conditions of jump formation which includes submerged jumps [8] and air entrainment in hydraulic jumps [9,10].

Scale effects are the result of distortions between scaled model measurements and prototype readings as a consequence of prototype constraints that are not correctly scaled to the miniature universe, which in turn leads to force ratios not scaling correctly between the model and prototype [11]. When carrying out experiments on large prototypes, effects such as surface tension, capillary forces and head loss due to channel bed roughness are negligible.

However, it is possible that these small forces can take precedence over gravity when producing the same experiments on a smaller scale [12]. As surface tension is regarded as insignificant for most prototypes in hydraulic engineering, however not potentially the case for small water depths [13], it is important to try to replicate the experiment in conditions as small as possible so that if a factor such as surface tension does affect the results, it will be noticeable.

The scale effect on the air entrainment process in a hydraulic jump was studied by Chanson and Chachereau [14] where after analyzing existing data, they concluded that, for hydraulic jumps with an upstream Froude number of 5.1, the void fraction data obtained with a Reynolds number less than 4 x 10^4 could not be scaled up to when the flow was at a Reynolds number of 1 x 10^5.

Further, they also concluded that in terms of parameters such as bubble count rate, turbulence, bubble chord time distributions and bubble cluster characteristics could not be scaled up with flows where Reynolds number was up to 1.25 x 10^5.

Peak all and Warburton [15] looked at the surface tension in small hydraulic river models and highlights the importance of considering the surface tension with dimensionless term Weber number used to ensure similarity. However, their attempts to come up with a critical Weber number to ensure dynamic similarity with the model and prototype was fraught with difficulties with issues in the definition of the Weber number where the ratio $V^2\rho l/\sigma$ is used at some places and the square root of the same ratio is used elsewhere.

Not many instances of studies of the scale effect on the hydraulic jump are found in the documented literature and hence identified as an area which requires further studies. Therefore, this study was carried out to determine the effect of scale on the on the well analyzed, predicted and documented behavior of the hydraulic jump phenomenon. This is also important as most of the scaled down physical models used to study hydraulic and other phenomena in engineering applications are getting increasingly smaller.

II. MATERIALS AND METHODS

To study the effect of scale on the hydraulic jump, an experimental approach was used which included creating a hydraulic jump on two flumes at the hydraulics and fluid mechanics laboratory at Nottingham Trent University, UK for different flows with details of the experimental setup shown as in Table 1 below. Full details of the experiments carried out along with all the raw data are available in Pashouros [16].

(a) 2.5 m flume used in experimental work.

International Journal of Science, Engineering and Technology

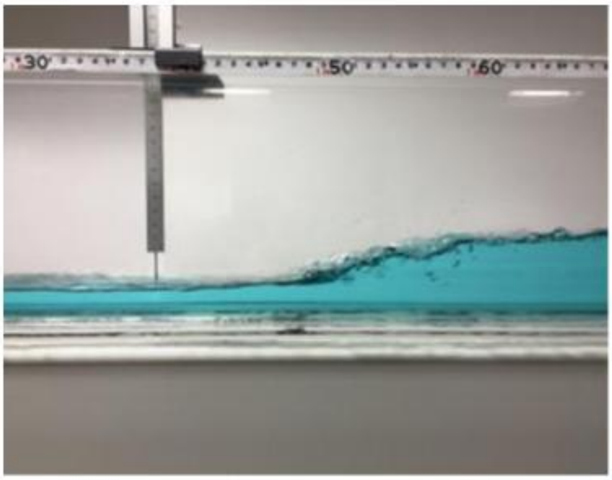
An Open Access Journal

(b) 5.0 m flume used in experimental work.
Fig 2. Flumes used in the experimental work, 2(a) shows the 2.5 m flume and 2(b) shows the 5mflume.

The depths of flow before and after the hydraulic jump were measured using the standard depth gauge and specific energy before and after the jump.

Table 1. Summary of the experiments carried out 3 (a) - Depth gauge used in measuring flow depth with the discharge measurement apparatus.

Flume details			Number of different sets of experiments carried out	Range of measured values for					
Flume	Length(mm)	Width(mm)		Flow rates (m3/s)	y1 (m)	y2 (m)	Es1 (m)	Es2 (m)	Lj (m)
Large	5000	78.5	53	0.001 -0.003	0.013 -0.030	0.039 -0.080	0.050 -0.120	0.032 -0.080	0.082 -1.220
Small	2500	55	34	0.00015 - 0.00068	0.004 -0.016	0.018 -0.045	0.020 -1.080	0.030 -0.072	0.110 -0.700



(a) Depth gauge used in measuring flow depth.

(b) 5m flume with the discharge measurement apparatus.
Fig 3. Depth gauge used in measuring flowdepth is shown in 3(a) and 5m flume with the discharge measurement apparatus is shown in 3(b).

Were measured with a Pitot Tube. Flow rates were obtained by the standard apparatus provided in the flume where a known mass of water was collected for a known time and the volumetric flow rate was computed as (mass / density of water) / time.

The depth of flow after the jump (y2) was calculated using the standard conjugate depth equation which gives the relationship before the depth of flow (y1) and the depth after the jump (y2), obtainable from the application of the momentum and continuity equations between a control volume which starts before the jump (suffixes 1) and after the jump(suffixes 2).

International Journal of Science, Engineering and Technology
An Open Access Journal

$$y_2 = \frac{y_1}{2}\left(\sqrt{1+8F_{r1}^2}-1\right)\ldots..\ (1)$$

Where Fr1 is the Froude number before the jump.

The specific energy head before and after the jump was calculated using the standard specific energy equation

$$E_{s1} = y_1 + \frac{v_1^2}{2g} \quad \ldots\ldots(2)$$

$$E_{s2} = y_2 + \frac{v_2^2}{2g} \quad \ldots\ldots(3)$$

The energy head loss during the jump was calculated as Es1 – Es2.

The length of the hydraulic jump (Lj) is a parameter which is so very hard to measure due to varying starting and ending points of the jump along with varying amounts of air entrainment leading to intensive turbulence [17, 18].

The roller length (Lr), which is the length of the recirculation zone, is considered by some researchers to be a better representation of the hydraulic jump length [19]. Different researchers have come up with different, empirical formulae to determine the hydraulic jump length usually for free, non-submerged jumps.

Of the different formulae by different researchers to determine hydraulic jump length, perhaps the most well known is the one by Hager [19] as given in equation 4.

$$\frac{L_j}{y_1} = 220\,Tanh\left(\frac{F_{r1}-1}{22}\right)\ldots\ldots\ (4)$$

Sylvester [20], came up with the following empirical equation using regression analysis on experimental data obtained on a channel with rough bed conditions.

$$\frac{L_j}{y_1} = 9.75(F_{r1}-1)^{1.01}\ldots\ldots\ \mathbf{(5)}$$

Gupta et al, [21] using Buckingham Pi theorems and regression analysis of experimental data came up with an expression to determine the length of the hydraulic jump shown in equation 6.

$$\frac{Lj}{y_1} = 4769.1\left[\frac{F_{r1}^{2.1}}{Re_1}\right] + 25.064\ldots\ldots\ \mathbf{(6)}$$

There are a few other equations researchers have come up with, all of which are based on fitting regression equations on experimental data and hence are empirical equations.

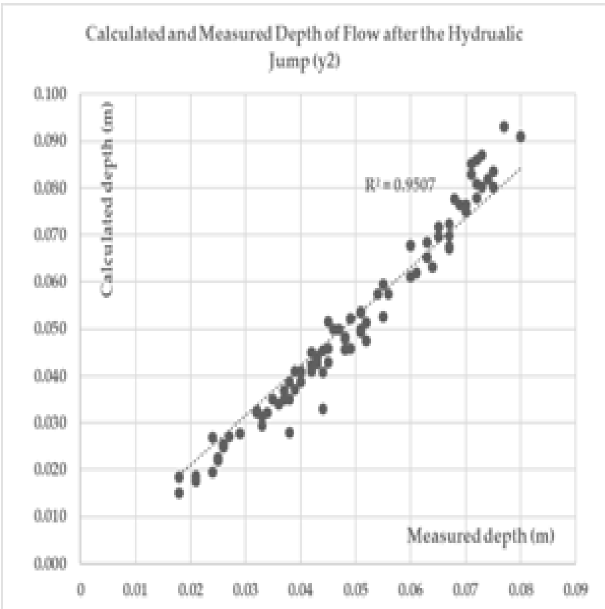
However, as the equations given above cover a wide variation of different parameters bed roughness; they have been used in this study.

All these calculated values of depth of flow after the hydraulic jump, height of the jump, specific energy before and after the jump and length of the jump were compared with the corresponding measured values for all the 87 sets of data obtained by creating a hydraulic jump for different flows for the larger, 5m flume (53 sets of data) and smaller, 2.5 m flume (34 sets of data).

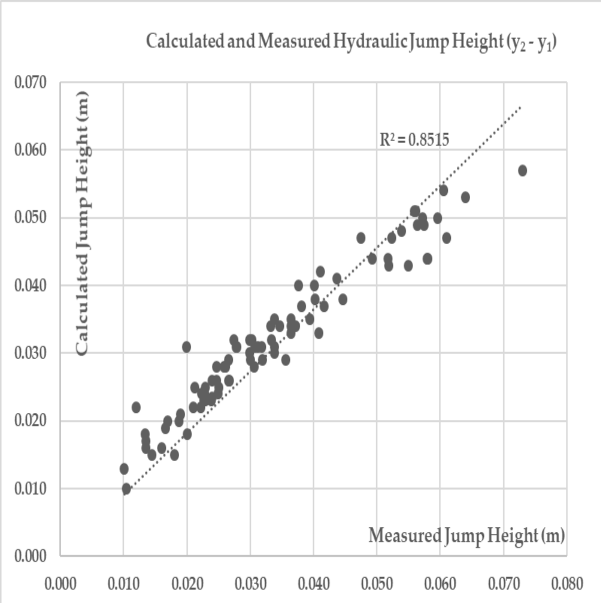
III. RESULTS AND DISCUSSION

The results obtained as per the methodology explained above are shown in Figures 4, 6 and 7 below. Figure 4(a) shows the comparison between the calculated and measured depth of flow after the hydraulic jump (y2) for all the 87 sets of data for bothflumes.

Figure 4(b) shows the comparison of calculated and measured hump height (ie y2 – y1), again for all the 87 sets of data for both flumes.



(a) Calculated and measured depth of flowafter the hydraulic jump.



(b) calculated and measured jump height.

Fig 4. Calculated and measured depth of flow after the hydraulic jump 4(a) and calculated and measured jump height 4(b).

As can be seen from the graphs in Figure 4(a) and 4(b), the agreement between measured and calculated values are excellent with coefficients of correlation of 0.95 and 0.85. This suggests that the conjugate depth equations hold true even at this scale and the effect of capillary actions and surface tension forces are either nonexistent or negligible.

Figure 5 shows the variation of calculated depth afterthe jump (y2) with the Reynolds number for the flow rates and velocities before the jump (Re1) used in the experiment. As can be seen, this relationship again shows a strong co-relation with a correlation Coefficient of 0.95 and this is somewhat different to the findings of Chanson and Chachereau [14].

However, the finding by these two authors related void fraction data, bubble count rate, turbulence, bubble chord time distributions and bubble cluster characteristics rather than the straightforward jump heights where one does not have to depend on any empirical formulae.

Figure 6 (a) shows the calculated (from equation 2 above) and measured specific energy head before the hydraulic jump and Figure 6(b) shows the calculated (from equation 3 above) and measured specific energy head after the hydraulic jump. Figure 6(c) shows the calculated and measured energy head loss in the hydraulic jump.

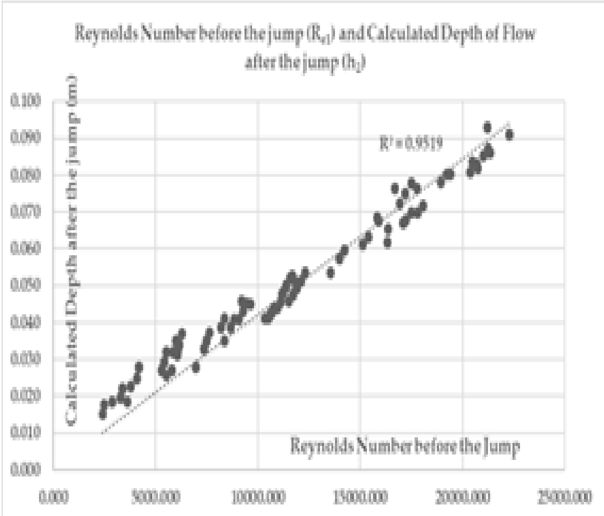
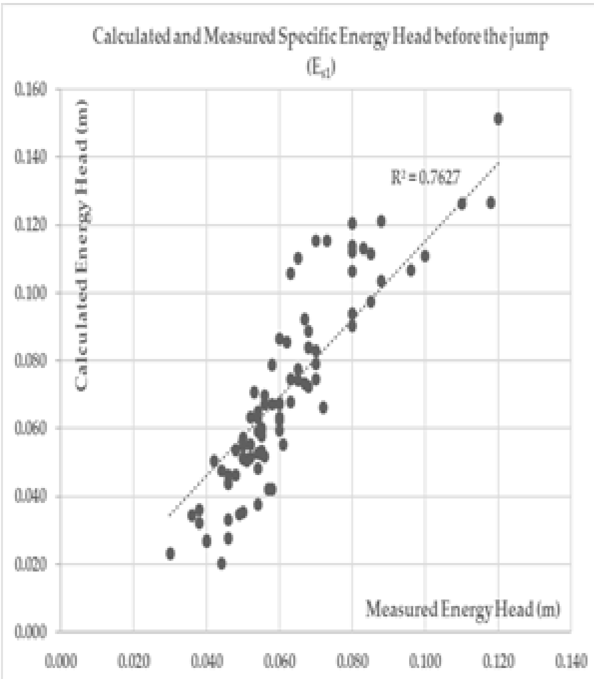
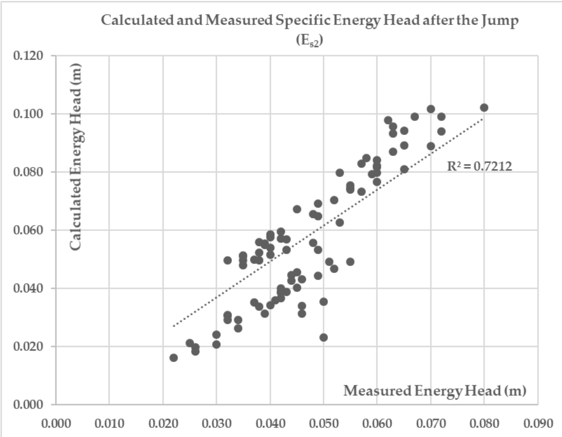


Fig 5. Variation of depth of flow after the jump (y2) with the Reynolds number before the jump (Re1).

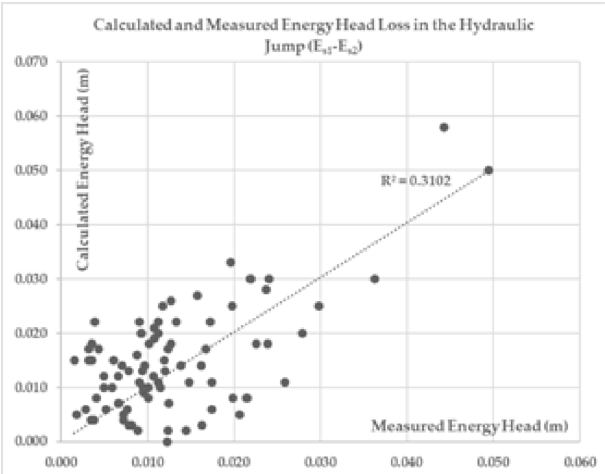
As seen from Figures 6(a), 6(b) and 6(c) the agreement between the calculated values and the corresponding measured values are quite acceptable for specific energy before and after the jump except perhaps for the energy loss comparison where the correlation coefficient is only 0.31. Unlike the depth of flow, the measurement of energy can a bit tricky as it was measured using a Pitot tube. However, within experimental error possible with the instruments that were available, the agreement can be considered adequate.



(a) Calculated and measured specific energyhead before the hydraulic jump.



(b) Calculated and measured specific energyhead after the hydraulic jump.

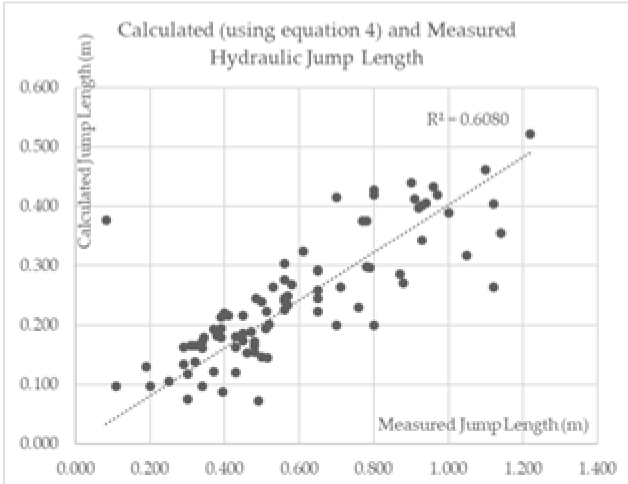


(c) Calculated and measured energy head loss.

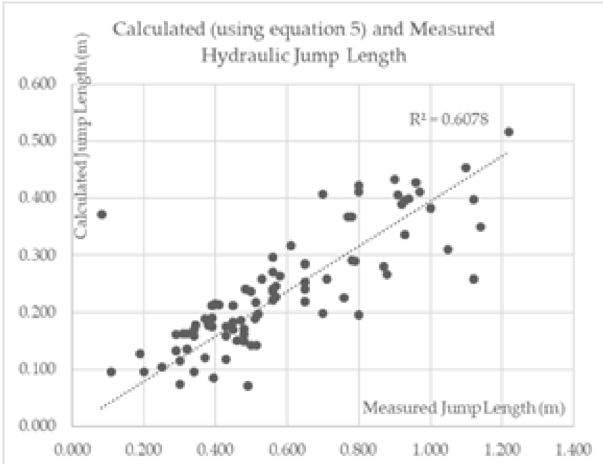
Fig 6. Calculated and measured specific energy head before the hydraulic jump 6(a), calculated and measured specific energy head after the hydraulic jump 6(b) and calculated and measured energy head loss 6(c).

Figures 7(a), 7(b) and 7(c) show the comparison between the measured jump length and calculated jump length (Lj) using the equations 4, 5 and 6. As mentioned the length of the hydraulic jump is a parameter which is very difficult to measure due mainly to the turbulence and varying nature of starting and ending points of the jump.

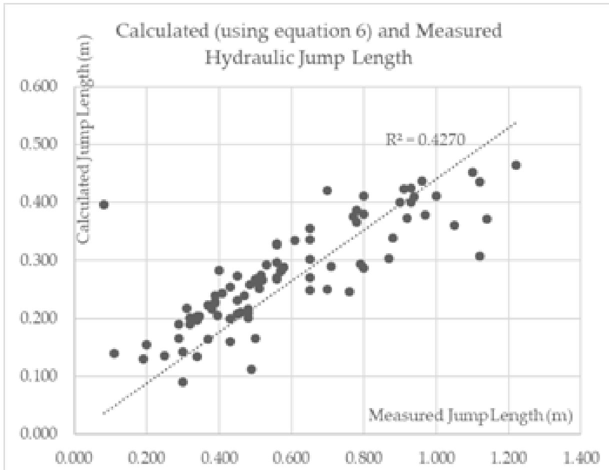
However, despite this difficulty, good agreements with the measured values and calculated values using equations 4 and 5 for the length of the hydraulic jump was obtained, whereas the same with equation 6 is acceptable. Its also worth noting that equations 4 and 5 calculated hydraulic jump lengths were almost identical, thus yielding a very similar correlation coefficient of 0.61.



(a) Comparison of calculated (using equations 4)and measured hydraulic jump length.



(b) Comparison of calculated (using equations 5)and measured hydraulic jump length.

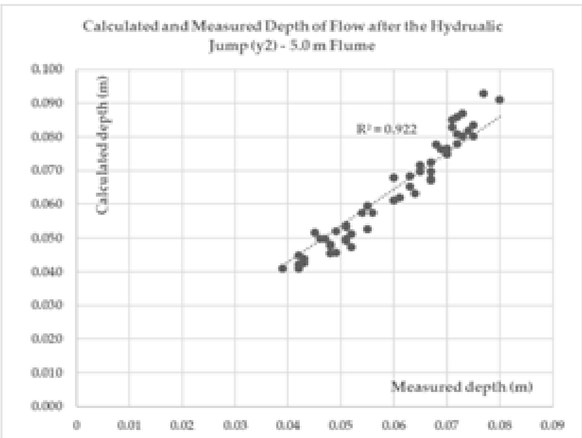


(c) Comparison of calculated (using equations 6)and measured hydraulic jump length.

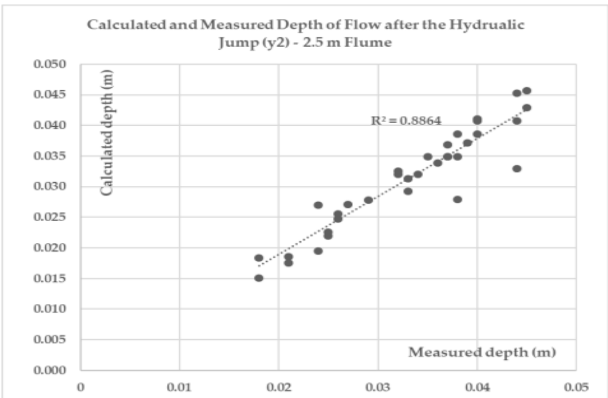
Fig 7. Comparison of calculated (using equations 4, 5 and 6 above) and measured hydraulic jump length.

Figure 8 shows the comparison of measured and calculated graphs for parameters for the two flumes separately. As can be seen, the agreement between the calculated and measured values is better for the larger 5m flumes in all parameters than for the same parameters for the smaller 2.5m flume. This may be due to surface tension and or capillary forces play according to Henderson's [22] suggestion that for channel depth and widths are an inch or two (0.025 –0.05 m).

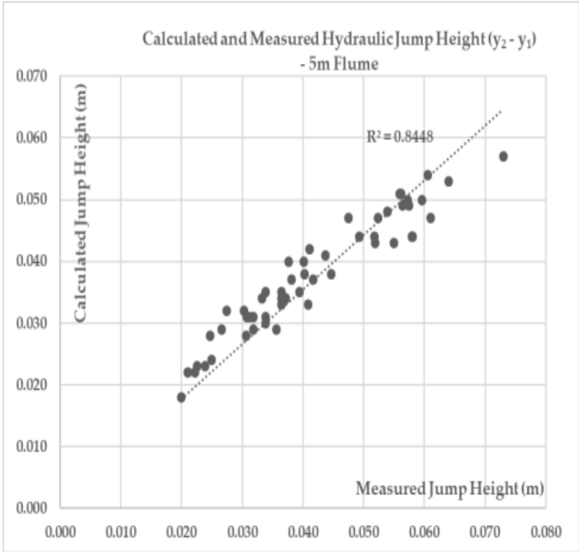
According to Novak et al [23] for shallow flows, the Weber number should be greater than 11 to avoid surface tension effects. The Weber number (calculated as $V^2\rho l/\sigma$) for the present study works out to be a minimum of 204 and a maximum of 7233 with an average of about 2658 and hence this yardstick suggests that surface tension forces are (likely to be insignificant in this study. What is clear however is that more studies are required before establishing the threshold values and critical Weber numbers for establishing the presence of surface tension forces and capillary forces in small scale hydraulic model studies?



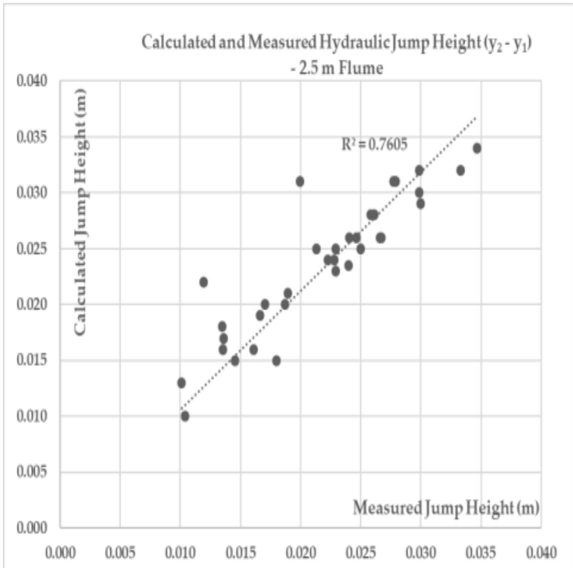
(a)



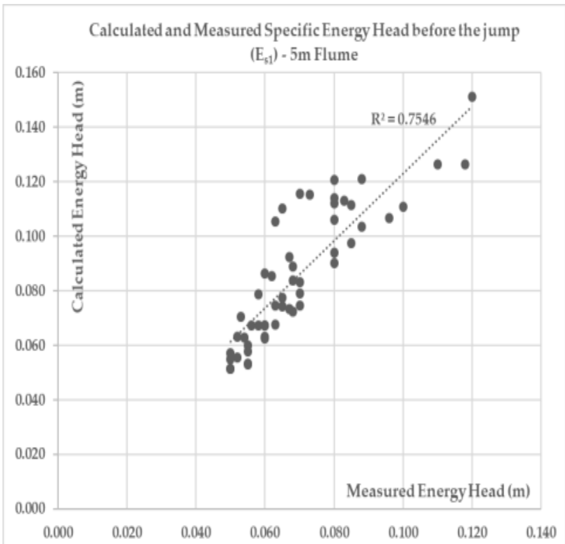
(b)



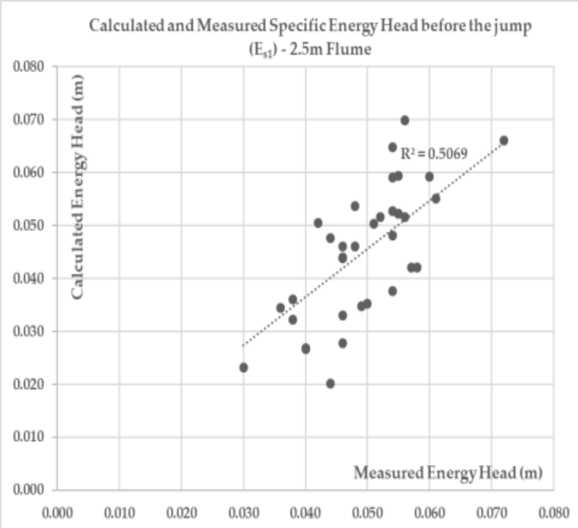
(c)



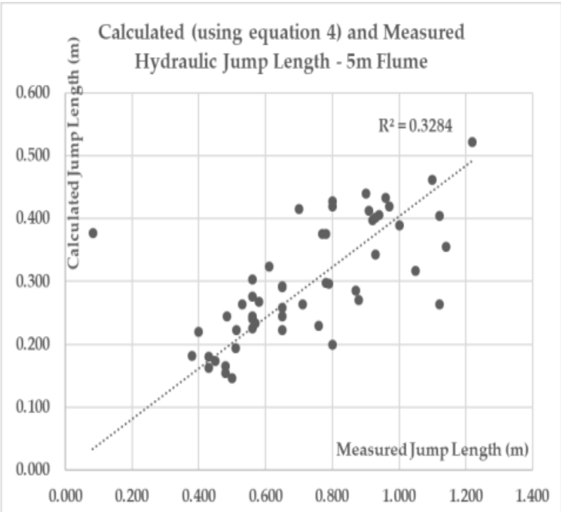
(d)



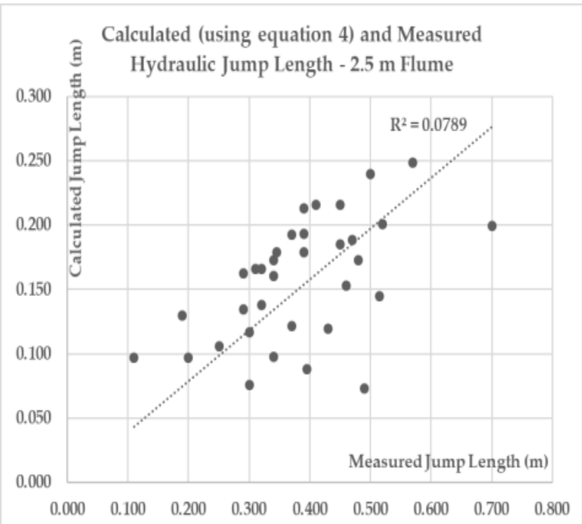
(e)



(f)



(g)



(h)

Fig 8. Comparison of measured and calculated graphs for parameters for the two different flumes of 5mand 2.5m.

IV. CONCLUSIONS

This study looked at the possible effects of forces which can affect the hydraulic phenomena at smaller scale such as the capillary forces and surface tension forces of a liquid. Specifically, the phenomenon that was looked at was the hydraulic jump and as seen from the results and ensuing discussion, it's apparent that the effects of these capillary forces and surface tension effects are either minimal or no existent altogether. It's also possible to conclude that the standard analytical methods used in the analysis of a hydraulic jump, which is the application of the continuity equation along with momentum equation, should yield very accurate results even at this small scale.

The study also throw light on the possibility of using small scale experimental apparatus (such as flumes) for hydraulic model studies which is an essential part in testing costly hydraulic structures such as dams without having to include parameters such as capillary and surface tension forces, though more studies on the same are recommended.

1. Abbreviations and Notations:

- y1 and y2 – depth of flow before the jump and afterthe jump respectively
- Es1 and Es2 - Specific energy before the jump andafter the jump respectively
- Lj – Length of the hydraulic jump
- Fr1 – Froude number before the jump
- Re1 – Reynolds number before the jump
- V – Velocity of flow
- ρ – Mass density of water
- L – Length parameter (for Weber number calculation in open channels flow depth is used as L)
- σ – Surface tension

REFERENCES

[1] Rajaratnam, N. (1965). The Hydraulic Jump as a Well Jet. J. Hydraul. Div. 91, 107–132.

[2] Hager, W.H.; Bremen, R.; Kawagoshi, N. (1990). Classical hydraulic jump: Length of roller. J. Hydraul. Res., 28, 591–608.

[3] Carollo, F.G.; Ferro, V.; Pampalone, V. (2009) New solution of classical hydraulic jump. J. Hydraul. Eng., 135, 527–531.

[4] Chanson, H. Development of the Bélanger equation and backwater equation by Jean-Baptiste Bélanger (1828). (2009) J. Hydraul. Eng., 135, 159–163.

[5] Hager, W.H. (1992). Energy Dissipators and Hydraulic Jump. Kluwer Academic, Dordrecht, the Netherlands. ISBN 0-7923-1508-1.

[6] Hassanpour, N., Hosseinzadeh Dalir, A., Farsadizadeh, D., & Gualtieri, C. (2017). An Experimental Study of Hydraulic Jump in a Gradually Expanding Rectangular Stilling Basin with Roughened Bed. Water, 9(12), 945. https://doi.org/10.3390/w9120945.

[7] Palermo, M., & Pagliara, S. (2017). A Review of Hydraulic Jump Properties on Both Smooth and Rough Beds in Sloping and Adverse Channels. Acta Scientiarum Polonorum Formatio Circumiectus, 1, 91–105. https://doi.org/10.1 5576/asp.fc/2017. 16.1.

[8] Ma F., Y. Hou & P. Prinos (2001) Numerical calculation of submerged hydraulic jumps, Journal of Hydraulic Research, 39:5, 493-503, DOI: 10.1080/00221686.2001.9628274

[9] Albarkani, M.S.S., Tan, L.W. and Al-Gheethi, A. (2018) 'Characteristics of Air Entrainment in Hydraulic Jump', IOP Conference Series: Earth and Environmental Science, 140 IOP Publishing, p. 012063.

[10] Chanson, H. (2011) 'Hydraulic jumps: turbulence and air bubble entrainment', La Houille Blanche,(3) EDP Sciences, pp. 5–16.

[11] Heller, V. (2011). Scale effects in physical hydraulic engineering models. Journal of Hydraulic Research, 49(3), 293–306.

[12] Glicksman, L. R., Hyre, M. R., & Farrell, P. A. (1994). Dynamic similarity in fluidization. International Journal of Multiphase Flow, 20, 331–386. https://doi.org/10.1016/0301-9322(94)90077-9

[13] Martin, H., Pohl, R, 2000. Technische Hydromechanik [Technical hydro mechanics]. Verlag fu¨r Bauwesen, Berlin (in German).

[14] Chanson, H. and Chachereau, Y., (2013). Scale effects affecting two-phase flow properties in hydraulic jump with small inflow Froude number. Experimental Thermal and Fluid Science, 45, pp.234-242.

[15] Peakall, J. and Warburton, J, (1996). Surface tension in small hydraulic river models- The significance of the Weber number. Journal of Hydrology, 35(2), pp.199-212.

[16] Pashouros, Lee (2017). The Scale Effect of Hydraulic Jump, Unpublished BSc (Hons) dissertation, Nottingham Trent University, Nottingham NG1 4FQ, United Kingdom.

[17] Carollo, F.G.; Ferro, V.; Pampalone, V. (2012). New Expression of the Hydraulic Jump Roller Length. J. Hydraul. Eng. 138, 995–999.

[18] Urbański, J., Siwicki, P., Kiczko, A., Kozioł, A., & Krukowski, M. (2018). The length of the hydraulic jump on the basis of physical and numerical modeling. Annals of Warsaw University of Life Sciences – SGGW. Land Reclamation, 50(1), 33–42. https://doi.org/ 10.2478/sggw-2018-0003

[19] Hager, W.H. and Bremen, R. (1989). Classical hydraulic jump: Sequent depths. J. Hydraul. Res. 27, 565–585

[20] Silvester, R. (1964). Hydraulic jump in all shapes of horizontal channels. J. Hydraul. Div. ASCE, 90: 23–55

[21] Gupta, S. K., Mehta, R. C., & Dwivedi, V. K. (2013). Modeling of Relative Length and Relative Energy Loss of Free Hydraulic Jump in Horizontal Prismatic Channel. Procedia Engineering, 51, 529–537. https://doi.org/ 10.1016/j.proeng.2013.01.075.

[22] Henderson, F. M. (1966). Open Channel Flow. London, England: Prentice-Hall.

[23] Novak, P., Moffat, A. I. B., Nalluri, C., & Narayanan, R. (2006). Hydraulic Structures (4th Ed.). London, England: CRC Press.